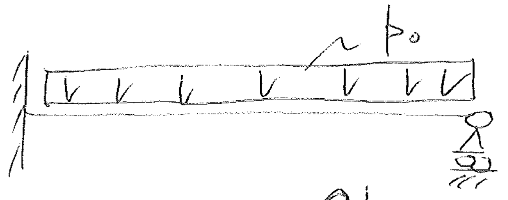
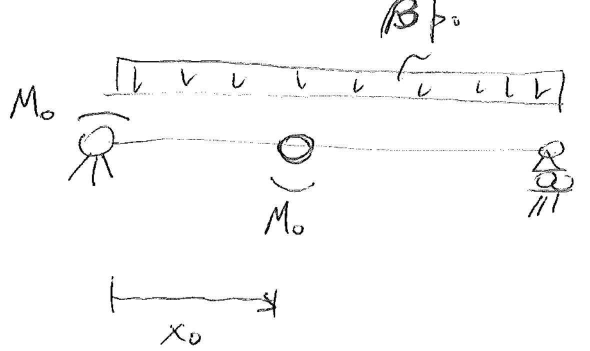
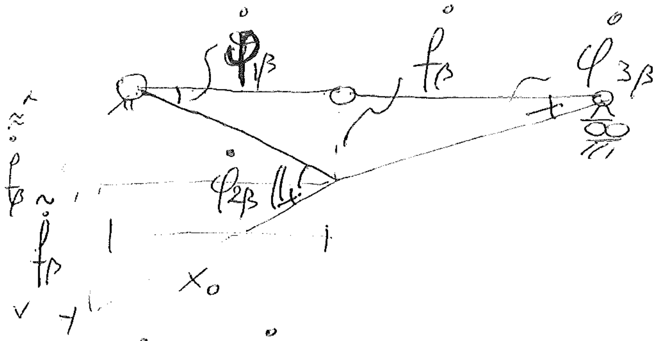
Calcolo moltiplicatore cinematico in forma parametrica (1)







$$\dot{\tilde f}_\beta = \dot\varphi_{3\beta} \cdot x_0 = \frac{\dot f_\beta}{l-x_0}\, x_0$$

$$\dot{\hat f}_\beta = \dot\varphi_{3\beta} \cdot l = \frac{\dot f_\beta}{l-x_0}\, l$$

$$\dot\varphi_{1\beta} = -\frac{\dot f_\beta}{x_0} \qquad \dot\varphi_{2\beta} = -\dot\varphi_1 + \dot\varphi_3 \qquad \dot\varphi_{3\beta} = \frac{\dot f_\beta}{l-x_0}$$

$$\dot\varphi_{2\beta} = \frac{\dot f_\beta}{x_0} + \frac{\dot f_\beta}{l-x_0} = \dot f_\beta\, \frac{\cancel{x_0} + l - \cancel{x_0}}{x_0(l-x_0)} = \dot f_\beta\, \frac{l}{x_0(l-x_0)}$$

$$\dot v_\beta(x) = \begin{cases} \dot f_\beta \cdot \dfrac{x}{x_0} & 0 \le x \le x_0 \\ \dfrac{\dot f_\beta\, l}{l-x_0}\left(1 - \dfrac{x}{l}\right) & x_0 \le x \le l \end{cases} \Rightarrow \begin{cases} \dot f_\beta\, \dfrac{x}{x_0} & 0 \le x \le x_0 \\ \dot f_\beta \left(\dfrac{l}{l-x_0} - \dfrac{x}{l-x_0}\right) & x_0 \le x \le l \end{cases}$$

$$\Rightarrow \begin{cases} \dot f_\beta\, \dfrac{x}{x_0} & 0 \le x \le x_0 \\ \dot f_\beta\, \dfrac{l-x}{l-x_0} & x_0 \le x \le l \end{cases}$$

$$\dot W_\beta = \int_0^l \beta p_0 \cdot \dot v_\beta(x)\, dx = \beta p_0 \int_0^{x_0} \dot f_\beta \frac{x}{x_0}\, dx + \beta p_0 \int_{x_0}^{l} \dot f_\beta \frac{l-x}{l-x_0}\, dx$$

$$= \beta p_0 \frac{\dot f_\beta}{x_0} \left[\frac{x^2}{2}\right]_0^{x_0} + \beta p_0 \frac{\dot f_\beta}{l-x_0} \left[l x - \frac{x^2}{2}\right]_{x_0}^{l}$$

$$= \beta p_0 \frac{\dot f_\beta}{x_0} \frac{x_0^2}{2} + \beta p_0 \frac{\dot f_\beta}{l-x_0} \left[l^2 - \frac{l^2}{2} - l x_0 + \frac{x_0^2}{2}\right]$$

$$\dot{W}_\beta = \beta p_0 \mathring{f}_\beta \left[\frac{x_0}{2} + \frac{\ell^2/2}{\ell - x_0} - \frac{\ell x_0}{\ell - x_0} + \frac{x_0^2/2}{(\ell - x_0)} \right]$$

$$= \beta p_0 \mathring{f}_\beta \left[\frac{x_0}{2} + \frac{1}{2}\frac{(\ell - x_0)^2}{(\ell - x_0)} \right] = \frac{\beta p_0 \mathring{f}_\beta}{2} \left[x_0 + \ell - x_0 \right]$$

$$\dot{W}_\beta = \beta \frac{p_0 \mathring{f}_\beta \ell}{2}$$

$$\dot{D}_\beta = -M_0 \cdot \mathring{\varphi}_{1\beta} + M_0 \cdot \mathring{\varphi}_{2\beta} = +M_0 \frac{\mathring{f}_\beta}{x_0} + M_0 \frac{\mathring{f}_\beta \ell}{x_0(\ell - x_0)}$$

$$= M_0 \mathring{f}_\beta \frac{\cancel{\ell} - x_0 + \cancel{\ell}}{x_0(\ell - x_0)} = M_0 \mathring{f}_\beta \frac{2\ell - x_0}{x_0(\ell - x_0)}$$

$$\beta = \frac{\dot{D}_\beta}{\frac{p_0 \mathring{f}_\beta \cdot \ell}{2}} = 2 \frac{M_0 \cancel{\mathring{f}_\beta} \frac{2\ell - x_0}{x_0(\ell - x_0)}}{p_0 \cancel{\mathring{f}_\beta} \ell} \qquad p_0 = \frac{M_0}{\ell^2}$$

$$\beta = 2 \frac{\cancel{M_0} \cdot \frac{2\ell - x_0}{x_0(\ell - x_0)}}{\frac{\cancel{M_0}}{\ell^{\cancel{2}}} \cdot \cancel{\ell}} = 2\ell \frac{2\ell - x_0}{x_0(\ell - x_0)}$$

$$\min_{x_0} \{\beta\} \Rightarrow \frac{d\beta}{dx_0} = 0 \qquad \frac{d}{dx_0}\left[2\ell \frac{2\ell - x_0}{x_0 \ell - x_0^2} \right] = 0$$

$<x_0<\ell$

$$= 2\ell \frac{-1(x_0 \ell - x_0^2) - (2\ell - x_0)(\ell - 2x_0)}{(x_0 \ell - x_0^2)^2} = 0$$

$$\Rightarrow -\cancel{x_0 \ell} + x_0^2 - 2\ell^2 + 4x_0 \ell + \cancel{x_0 \ell} - 2x_0^2 = 0$$

$$-x_0^2 + 4x_0 \ell - 2\ell^2 = 0$$

$$\left(\frac{x_0}{\ell}\right)^2 - 4\left(\frac{x_0}{\ell}\right) + 2 = 0$$

$$\frac{x_0}{\ell} = \frac{2 \pm \sqrt{4 - 2}}{1} = 2 \pm \sqrt{2} \Rightarrow x_0 = \begin{cases} (2+\sqrt{2})\ell > \ell \,! \\ (2-\sqrt{2})\ell = 0.58579\,\ell \end{cases}$$

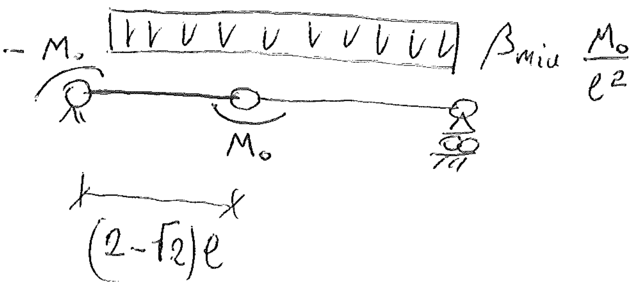
$$\min \beta = 2l\,\frac{2l-(2l-\sqrt{2}l)}{(2l-\sqrt{2}l)(l-2l+\sqrt{2}l)} = 2l\,\frac{\cancel{2l}-\cancel{2l}+\sqrt{2}l}{(2l-\sqrt{2}l)(-l+\sqrt{2}l)}$$

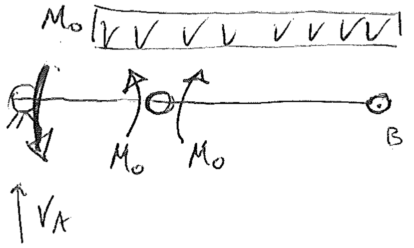
$$= 2l^2\,\frac{\sqrt{2}}{l^2(2-\sqrt{2})(-1+\sqrt{2})} = 2\,\frac{\sqrt{2}}{(-2+\sqrt{2}+2\sqrt{2}-2)} = \frac{2\sqrt{2}}{(3\sqrt{2}-4)}$$

$$= 2\,\frac{\sqrt{2}\,(3\sqrt{2}+4)}{9\cdot 2-16} = 2\left(\frac{6+4\sqrt{2}}{2}\right) = 2(3+2\sqrt{2}) = 2(5.828)$$

$$= 11.6569 = \beta_{min}$$

$$\beta\Big|_{l/2} = 2l\,\frac{2l-l/2}{\frac{l}{2}\left(l-\frac{l}{2}\right)} = 2l^2\cdot\frac{\frac{3}{2}}{\frac{l^2}{4}} = 8\cdot\frac{3}{2} = 12. \quad (>\beta_{min})$$





$$\Sigma M_i\Big|_B = 0 \qquad V_A\cdot l - M_0 - \beta_{min}\frac{M_0}{l^2}\cdot\frac{l^2}{2} = 0$$

$$V_A = \left(1+\frac{\beta_{min}}{2}\right)\frac{M_0}{l}$$

$$V_A = \left[1+(3+2\sqrt{2})\right]\frac{M_0}{l} = (4+2\sqrt{2})\frac{M_0}{l}$$

$$M_\beta(x) = (4+2\sqrt{2})\frac{M_0}{l}x - M_0 - \beta_{min}\frac{M_0}{l^2}\cdot\frac{x^2}{2}$$

$$M_\beta(x) = -M_0\left[1-(4+2\sqrt{2})\frac{x}{l}+(3+2\sqrt{2})\left(\frac{x}{l}\right)^2\right]$$

$$M_\beta(x) = -M_0\left[1-2(2+\sqrt{2})\frac{x}{l}+(3+2\sqrt{2})\left(\frac{x}{l}\right)^2\right]$$

$$\max M_\beta \Rightarrow \frac{d}{dx}M_\beta = 0 \Rightarrow -(2+\sqrt{2})+(3+2\sqrt{2})\frac{x}{l} = 0 \qquad \frac{x}{l} = \frac{(2+\sqrt{2})(3-2\sqrt{2})}{(3+2\sqrt{2})(3-2\sqrt{2})}$$

$$\frac{x}{l} = \frac{6-4\sqrt{2}+3\sqrt{2}-4}{9-8} \Rightarrow x = (2-\sqrt{2})\,l$$

$$\max M_\beta = M_\beta \Big|_{x=2-\sqrt{2}\,\ell} = -M_0 \left[1 - 2(2+\sqrt{2})(2-\sqrt{2}) + (3+2\sqrt{2})(2-\sqrt{2})^2 \right]$$

$$= -M_0 \left[1 - 2(4-2) + (3+2\sqrt{2})(4 - 4\sqrt{2} + 2) \right]$$

$$= -M_0 \left[1 - 2(4-2) + (3+2\sqrt{2})(6-4\sqrt{2}) \right]$$

$$= -M_0 \left[1 - 2(4-2) + 2(3+2\sqrt{2})(3-2\sqrt{2}) \right]$$

$$= -M_0 \left[1 - 4 + 2(9-8) \right] = -M_0 \left[1 - 4 + 2 \right]$$

$$= -M_0 \left[-4 + 3 \right] = M_0 \qquad \text{qed.}$$