

CORSO DI T. E P. DI COSTRUZIONI E STRUTTURE

A.A. 2003-04

Seconda esercitazione scritta del 03.06.2004

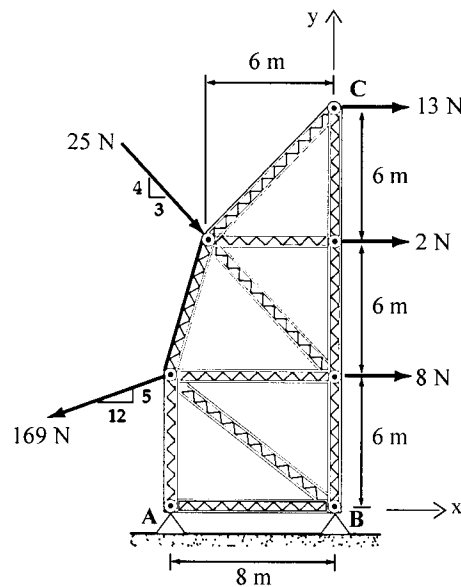
(da riconsegnare entro le 17:30 del 10.06.2004)

Nota: I risultati numerici (in forma frazionaria o con 3 cifre decimali) vanno riportati su questo stesso foglio, nei riquadri predisposti; i calcoli (in forma ordinata) vanno allegati su fogli in formato A4 bianchi o a quadretti.

Allievo:..... Matricola:.....

Esercizio n.1 (6 punti)

Si consideri la struttura data in figura



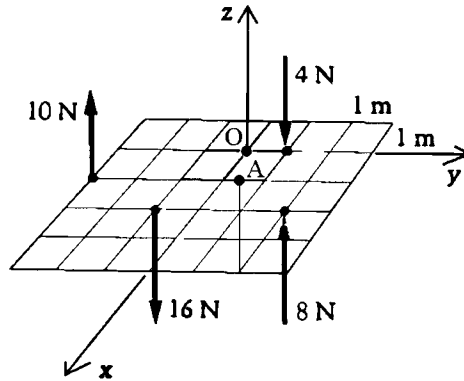
Calcolare la risultante \mathbf{F} delle 5 forze applicate, il suo modulo F e l'intersezione della retta a cui appartiene il vettore risultante con la retta x passante per AB e la retta y passante per BC.

$\mathbf{F} = \dots\dots\dots$; $F = |\mathbf{F}| = \dots\dots\dots$;

$x_{AB} = \dots\dots\dots$; $y_{BC} = \dots\dots\dots$;

Esercizio n.2 (6 punti)

Le 4 forze rappresentate in figura hanno linee di azione verticale ed agiscono su una griglia orizzontale di passo unitario.

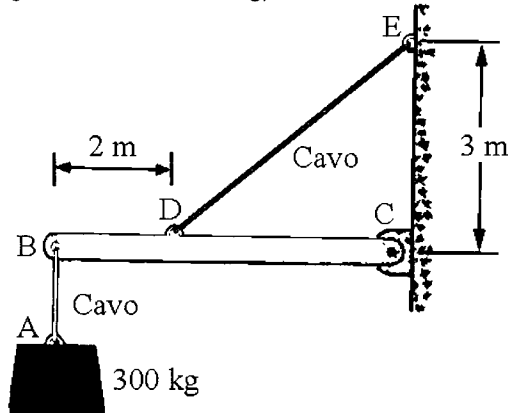


Per tale sistema di forze trovare la singola forza \mathbf{R} equipollente, il suo punto di applicazione (x_R, y_R) sulla griglia, il vettore \mathbf{M}_O momento risultante del sistema di forze rispetto all'origine O del sistema di riferimento Oxyz ed il vettore \mathbf{M}_A momento risultante della forza \mathbf{R} rispetto al punto $A = (4, 2, 2)$

$\mathbf{R} = \dots\dots\dots; (x_R, y_R) = \dots\dots\dots;$
$\mathbf{M}_O = \dots\dots\dots; \mathbf{M}_A = \dots\dots\dots;$

Esercizio n.3 (6 punti)

Un blocco di massa pari a 300 kg è sorretto tramite il cavo verticale AB dalla barra orizzontale BC lunga 6m e di peso trascurabile. La barra BC è collegata alla parete tramite la cerniera in C, che garantisce una reazione orizzontale H_C ed una verticale V_C , ed il cavo DE.

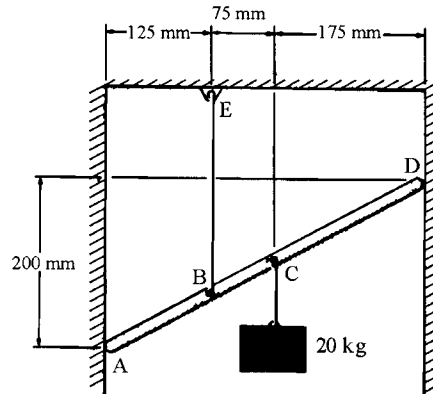


Si chiede di calcolare la tensione del cavo T_{DE} e le reazioni $H_{C,1}$ e $V_{C,1}$.
 Si chiede inoltre di calcolare la tensione del cavo T_{BE} e le reazioni $H_{C,2}$ e $V_{C,2}$ nel caso che il cavo diagonale fosse vincolato in E ed in B.

$T_{DE} = \dots\dots\dots; H_{C,1} = \dots\dots\dots; V_{C,1} = \dots\dots\dots;$
$T_{BE} = \dots\dots\dots; H_{C,2} = \dots\dots\dots; V_{C,2} = \dots\dots\dots;$

Esercizio n.4 (6 punti)

La barra leggera AD è sospesa per mezzo del cavo BE e sostiene, nel punto C, un blocco di massa $m = 20 \text{ kg}$. Le estremità A e D della barra sono in contatto con le pareti verticali in assenza attrito.

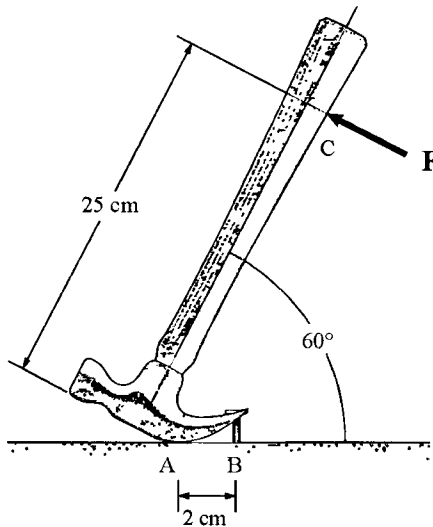


Determinare la tensione T_{BE} nel cavo BE e le reazioni (orizzontali) H_A e H_D all'estremità della barra.

$T_{DE} = \dots\dots\dots; H_{C,1} = \dots\dots\dots; V_{C,1} = \dots\dots\dots;$

Esercizio n.5 (6 punti)

Si assuma che la forza esercitata dal martello sulla testa del chiodo sia verticale e che il peso del martello risulti trascurabile. La forza F esercitata sul martello nel punto C ha modulo $F = 20 \text{ N}$ ed è diretta ortogonalmente al manico del martello stesso.

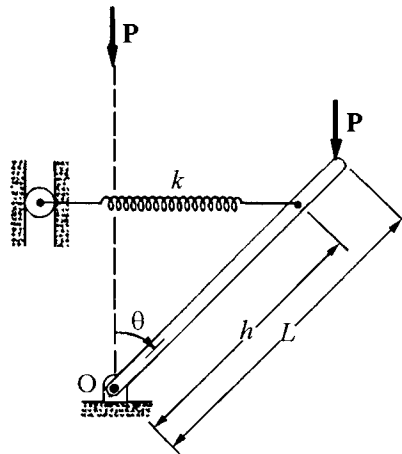


Calcolare la forza R esercitata dal martello sulla testa del chiodo e le reazioni orizzontale, H_A , e verticale, V_A , nel punto A.

$R = \dots\dots\dots; H_A = \dots\dots\dots; V_A = \dots\dots\dots;$
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Esercizio n.6 (bonus, 3 punti)

Una barra leggera rigida è fissata al terreno nel punto O tramite una cerniera che fornisce una reazione orizzontale H_O ed una verticale V_O .
 La molla di costante elastica k è vincolata a rimanere orizzontale ed è nella posizione di riposo quando $\theta = 0$.



Se $P L < k h^2$ esiste una configurazione di equilibrio per $0 < \theta < \pi/2$.
 Trovare l'angolo $\theta \neq 0$ tale per cui viene garantito l'equilibrio e la forza F_{el} nella molla in tale configurazione.
 Per $\theta = 0$ è possibile garantire l'equilibrio?

$\theta = \dots\dots\dots$; $F_{el} = \dots\dots\dots$;
Equilibrio ($\theta = 0$) <input type="checkbox"/> SI <input type="checkbox"/> NO

1	
2	
3	
4	
5	
6	
TOT	

II ESERCITAZIONE SCRITTA DEL 03.06.04

$$\textcircled{1} \quad \vec{F}_1 = +8\vec{L} \quad N$$

$$P_1 = (0, 6) \quad m$$

$$\vec{F}_2 = +2\vec{L} \quad N$$

$$P_2 = (0, 12) \quad m$$

$$\vec{F}_3 = +13\vec{L} \quad N$$

$$P_3 = (0, 18) \quad m$$

$$\vec{F}_4 = 25 \cdot \frac{3}{5}\vec{L} - 25 \cdot \frac{4}{5}\vec{j} = 15\vec{L} - 20\vec{j} \quad N$$

$$P_4 = (-6, 12) \quad m$$

$$\vec{F}_5 = -169 \cdot \frac{12}{13}\vec{L} - 169 \cdot \frac{5}{13}\vec{j} = -156\vec{L} - 65\vec{j} \quad N$$

$$P_5 = (-8, 6) \quad m$$

$$B = (0, 0) \quad m$$

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5$$

$$\vec{R} = (8+2+13+15-156)\vec{L} + (-20-65)\vec{j} \quad N$$

$$\vec{R} = -118\vec{L} - 85\vec{j} \quad N$$

$$|\vec{R}| = \sqrt{118^2 + 85^2} \quad N = \sqrt{21149}$$

$$|\vec{R}| = 145.427 \quad N$$

$$\vec{M}_{(B)} = (P_1 - B) \wedge \vec{F}_1 + (P_2 - B) \wedge \vec{F}_2 + (P_3 - B) \wedge \vec{F}_3 + (P_4 - B) \wedge \vec{F}_4 + (P_5 - B) \wedge \vec{F}_5$$

$$\vec{M}_{(B)} = 6\vec{j} \wedge 8\vec{L} + 12\vec{j} \wedge 2\vec{L} + 18\vec{j} \wedge 13\vec{L} + (-6\vec{L} + 12\vec{j}) \wedge (15\vec{L} - 20\vec{j}) + (-8\vec{L} + 6\vec{j}) \wedge (-156\vec{L} - 65\vec{j}) \quad N \cdot m$$

$$\vec{M}_{(B)} = 48(\vec{j} \wedge \vec{L}) + 24(\vec{j} \wedge \vec{L}) + 234(\vec{j} \wedge \vec{L}) + 120(\vec{L} \wedge \vec{j}) + 180(\vec{j} \wedge \vec{L}) + 520(\vec{L} \wedge \vec{j}) - 936(\vec{j} \wedge \vec{L}) \quad N \cdot m$$

$$\vec{M}_{(B)} = (48+24+234+180-936)(\vec{j} \wedge \vec{L}) + (120+520)\vec{L} \wedge \vec{j} \quad N \cdot m$$

$$\text{Ora, } \vec{j} \wedge \vec{L} = -\vec{k} \quad ; \quad \vec{L} \wedge \vec{j} = +\vec{k}$$

$$\vec{M}_{(B)} = -450(\vec{j} \wedge \vec{L}) + 640(\vec{L} \wedge \vec{j}) \quad N \cdot m = (450+640)(\vec{L} \wedge \vec{j}) \quad N \cdot m = 1090\vec{k} \quad N \cdot m$$

$$\vec{M}_{(B)} = (P_{AB} - B) \wedge \vec{R}$$

$$\text{dove } P_{AB} = (x_{AB}, 0)$$

sicché

$$\vec{M}_{(B)} = x_{AB} \vec{L} \wedge (-118\vec{L} - 85\vec{j}) \quad N \cdot m$$

$$\vec{M}_{(B)} = -85 x_{AB} (\vec{L} \wedge \vec{j}) \quad N \cdot m = -85 x_{AB} \vec{k} \quad N \cdot m$$

$$\Rightarrow 1090 \vec{k} = -85 x_{AB} \vec{k} \quad x_{AB} = -\frac{1090}{85} = -12.824 \quad m$$

①

$$\vec{M}_{(B)} = (P_{BC} - B) \wedge \vec{R}$$

dove $P_{BC} \equiv (0, y_{BC})$

sicché

$$\vec{M}_{(B)} = y_{BC} \vec{j} \wedge (-118\vec{i} - 85\vec{j}) \text{ N.m}$$

$$\vec{M}_{(B)} = -118 y_{BC} (\vec{j} \wedge \vec{i}) \text{ N.m} = -118 y_{BC} (-\vec{k}) \text{ N.m} = 118 y_{BC} \vec{k} \text{ N.m}$$

$$\Rightarrow 118 y_{BC} \vec{k} = 1090 \vec{k} \quad y_{BC} = \frac{1090}{118} = +9.237 \text{ m.}$$

②

$$\vec{F}_1 = 10 \vec{k} \text{ N}$$

$$P_1 \equiv (1, -3, 0) \text{ m}$$

$$\vec{F}_2 = -16 \vec{k} \text{ N}$$

$$P_2 \equiv (2, -1, 0) \text{ m}$$

$$\vec{F}_3 = +8 \vec{k} \text{ N}$$

$$P_3 \equiv (2, 2, 0) \text{ m}$$

$$\vec{F}_4 = -4 \vec{k} \text{ N}$$

$$P_4 \equiv (0, 1, 0) \text{ m}$$

$$\vec{M}_{(O)} = (P_1 - O) \wedge \vec{F}_1 + (P_2 - O) \wedge \vec{F}_2 + (P_3 - O) \wedge \vec{F}_3 + (P_4 - O) \wedge \vec{F}_4$$

$$\vec{M}_{(O)} = (\vec{i} - 3\vec{j}) \wedge 10\vec{k} + (2\vec{i} - \vec{j}) \wedge (-16\vec{k}) + (2\vec{i} + 2\vec{j}) \wedge 8\vec{k} + \vec{j} \wedge (-4\vec{k}) \text{ N.m}$$

$$\vec{M}_{(O)} = 10(\vec{i} \wedge \vec{k}) - 30(\vec{j} \wedge \vec{k}) - 32(\vec{i} \wedge \vec{k}) + 16(\vec{j} \wedge \vec{k}) + 16(\vec{i} \wedge \vec{k}) + 16(\vec{j} \wedge \vec{k}) - 4(\vec{j} \wedge \vec{k}) \text{ N.m}$$

$$\vec{M}_{(O)} = (10 - 32 + 16)(\vec{i} \wedge \vec{k}) + (-30 + 16 + 16 - 4)(\vec{j} \wedge \vec{k}) \text{ N.m}$$

e poiché $\vec{i} \wedge \vec{k} = -\vec{j}$; $\vec{j} \wedge \vec{k} = \vec{i}$

$$\vec{M}_{(O)} = (-6)(-\vec{j}) + (-2)\vec{i} = -2\vec{i} + 6\vec{j} \text{ N.m}$$

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$

$$\vec{R} = (10 - 16 + 8 - 4)\vec{k} \text{ N} = (-2)\vec{k} \text{ N} = -2\vec{k} \text{ N}$$

$$\vec{M}_{(O)} = (P_R - O) \wedge \vec{R} \quad \text{dove } P_R \equiv (x_R, y_R, 0)$$

$$\vec{M}_{(O)} = (x_R \vec{i} + y_R \vec{j}) \wedge (-2\vec{k}) \text{ N.m} = -2x_R (\vec{i} \wedge \vec{k}) - 2y_R (\vec{j} \wedge \vec{k}) \text{ N.m}$$

$$\vec{M}_{(O)} = -2x_R (-\vec{j}) - 2y_R \vec{i} \text{ N.m}$$

$$\Rightarrow -2\vec{i} + 6\vec{j} = -2y_R \vec{i} + 2x_R \vec{j} \quad \begin{cases} -2 = -2y_R \Rightarrow y_R = 1 \text{ m} \\ 6 = 2x_R \Rightarrow x_R = 3 \text{ m} \end{cases}$$

$$P_R \equiv (3, 1, 0) \text{ m.}$$

②

$$\vec{M}_{(A)} = (\vec{r} - \vec{A}) \wedge \vec{R}$$

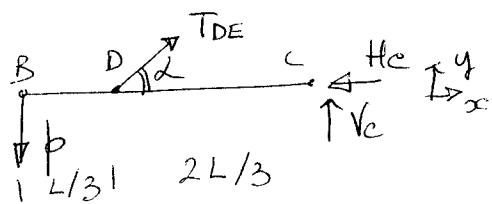
$$A = (4, 2, 2) \text{ m}$$

$$\vec{M}_{(A)} = (-\vec{i} - \vec{j} - 2\vec{k}) \wedge (-2\vec{k}) \quad \text{N.m}$$

$$\vec{M}_{(A)} = 2(\vec{i} \wedge \vec{k}) + 2(\vec{j} \wedge \vec{k}) \quad \text{N.m}$$

$$\vec{M}_{(A)} = 2\vec{i} - 2\vec{j} \quad \text{N.m}$$

③ 1. Diagramma di corpo libero:



$$p = 300 \text{ kp} \cdot 9.81 \text{ m/s}^2 = 2943 \text{ N}$$

$$\cos \alpha = \frac{4}{5}; \quad \sin \alpha = \frac{3}{5}$$

$$\rightarrow R_x = 0 \quad + T_{DE} \cos \alpha - H_c = 0$$

$$\uparrow R_y = 0 \quad -p + T_{DE} \sin \alpha + V_c = 0$$

$$\curvearrowright M_{z(c)} = 0 \quad pL - T_{DE} \sin \alpha \cdot \frac{2}{3}L = 0$$

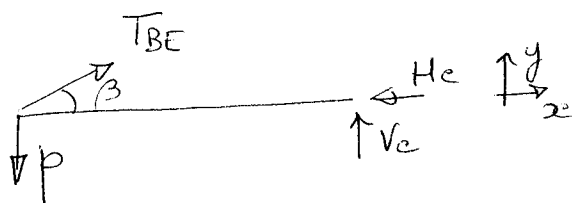
Risolvendo si ottiene:

$$T_{DE} = \frac{3p}{2 \sin \alpha} = \frac{3 \cdot 5}{2 \cdot 3} p = \frac{5}{2} p = 7357.5 \text{ N}$$

$$H_c = T_{DE} \cdot \cos \alpha = \frac{5}{2} p \cdot \frac{4}{5} = 2p = 5886.0 \text{ N}$$

$$V_c = p - T_{DE} \cdot \sin \alpha = p - \frac{5}{2} p \cdot \frac{3}{5} = -\frac{p}{2} = -1471.5 \text{ N}$$

2 Diagramma di corpo libero



$$\cos \beta = \frac{6}{\sqrt{45}} = \frac{2\sqrt{5}}{5}$$

$$\sin \beta = \frac{3}{\sqrt{45}} = \frac{\sqrt{5}}{5}$$

$$\rightarrow R_x = 0 \quad T_{BE} \cdot \cos \beta - H_c = 0$$

$$\uparrow R_y = 0 \quad -p + T_{BE} \sin \beta + V_c = 0$$

$$\curvearrowright M_{z(c)} = 0 \quad pL - T_{BE} \sin \beta L = 0$$

La soluzione è, in questo caso:

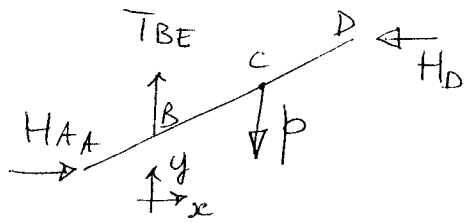
$$T_{BE} = \frac{pk}{k \sin \beta} = \frac{5p}{\sqrt{5}} = \sqrt{5}p = 6580.748 \text{ N}$$

③

$$H_c = T_{BE} \cdot \cos\beta = \sqrt{5}p \cdot \frac{2\sqrt{5}}{5} = 2p = 5886.0 \text{ N}$$

$$V_c = p - T_{BE} \cdot \sin\beta = p - \sqrt{5}p \cdot \frac{\sqrt{5}}{5} = p - p = 0 \text{ N}$$

④ Diagramma di corpo libero:



$$p = 20 \text{ kg} \cdot 9.81 \text{ m/s}^2 = 196.2 \text{ N}$$

$$\rightarrow R_x = 0 \quad H_A - H_D = 0$$

$$\uparrow R_y = 0 \quad T_{BE} - p = 0$$

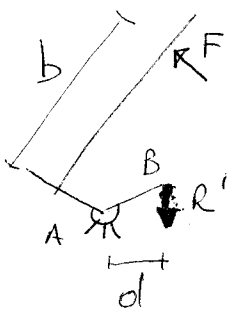
$$\curvearrowright M_{Z(A)} = 0 \quad T_{BE} \cdot 125 - p \cdot 200 + H_D \cdot 200 = 0$$

Si ottiene: $T_{BE} = p = 196.2 \text{ N}$

$$H_D = p - T_{BE} \cdot \frac{125}{200} = p \left(1 - \frac{5}{8}\right) = \frac{3}{8}p = 73.575 \text{ N}$$

$$H_A = H_D = 73.575 \text{ N}$$

⑤ Schema statico:



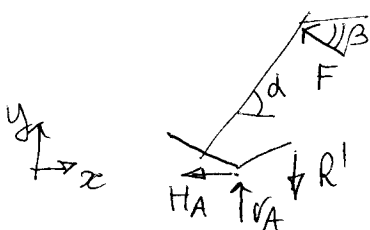
$$M_{Z(A)} = 0 \quad Fb - R'd = 0$$

$$R' = F \frac{b}{d} = 20 \cdot \frac{25}{2} \text{ N} \cdot \text{cm}$$

$$R' = 250 \text{ N}$$

NB: R' è la azione che il chiodo esercita sul martello, eguale e opposta a quella che il martello esercita sul chiodo.

Diagramma di corpo libero:



$$\beta = \frac{\pi}{2} - \alpha = 30^\circ$$

$$\rightarrow R_x = 0 \quad -F \cos\beta - H_A = 0$$

$$\uparrow R_y = 0 \quad V_A - R' + F \sin\beta = 0$$

④

$$H_A = -F \cos 30^\circ = -\frac{\sqrt{3}}{2} F = 17,321 \text{ N}$$

$$V_A = R' - F \sin 30^\circ = 250 - 10 \text{ N} = 240 \text{ N}$$

⑥

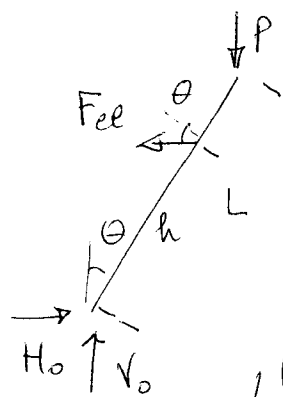


Diagramma di corpo libero

$$\rightarrow R_x = 0 \quad H_0 - F_{el} = 0$$

$$\uparrow R_y = 0 \quad V_0 - P = 0$$

$$\circlearrowleft M_{z(O)} = 0 \quad F_{el} \cos \theta \cdot h - P \sin \theta \cdot L = 0 \quad [7]$$

L'ultima è eq. pura; le prime 2 determinano le reazioni vincolari H_0, V_0 .

Si ha che $F_{el} = k \cdot \Delta x \quad \Delta x = h \sin \theta$

$$\Rightarrow F_{el} = k h \sin \theta$$

e per avere equilibrio, in base alla [7] deve risultare:

$$k h^2 \sin \theta \cos \theta - P \sin \theta = 0$$

Le soluzioni sono esprimibili come:

$$(k h^2 \cos \theta - P) \sin \theta = 0 \quad \left\{ \begin{array}{l} \sin \theta = 0 \rightarrow \theta = 0 \text{ o } \theta = \pi \\ \cos \theta = \frac{P}{k h^2} \rightarrow \theta = \arccos \frac{P}{k h^2} \end{array} \right.$$

si noti che l'ultima soluzione esiste solo a condizione che

$$\cos \theta = \frac{P}{k h^2} < 1 \quad \Rightarrow \quad P < k h^2$$

La prima soluzione esiste qualunque sia P : in questo caso infatti $F_{el} = 0$ e la forza P è allineata con la cerniera, sicché $M_{z(O)} = 0 \quad \forall P$.

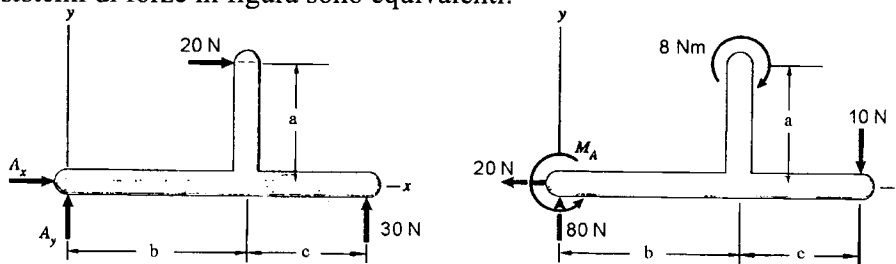
Nota: I risultati numerici (in forma frazionaria o con 3 cifre decimali) vanno riportati su questo stesso foglio, nei riquadri predisposti.

Allievo:..... Matricola:.....

FILA 1

Esercizio n.1 (6 punti)

I due sistemi di forze in figura sono equivalenti:

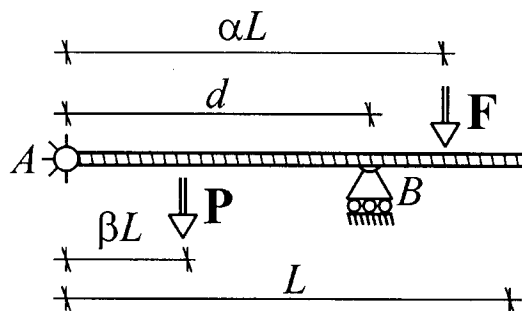


Siano $b = 600 \text{ mm}$, $a = c = 400 \text{ mm}$.
 Determinare le forze A_x , A_y e la coppia M_A .

$A_x = \dots\dots\dots$; $A_y = \dots\dots\dots$; $M_A = \dots\dots\dots$

Esercizio n.2 (6 punti)

Si consideri l'asta caricata in figura:

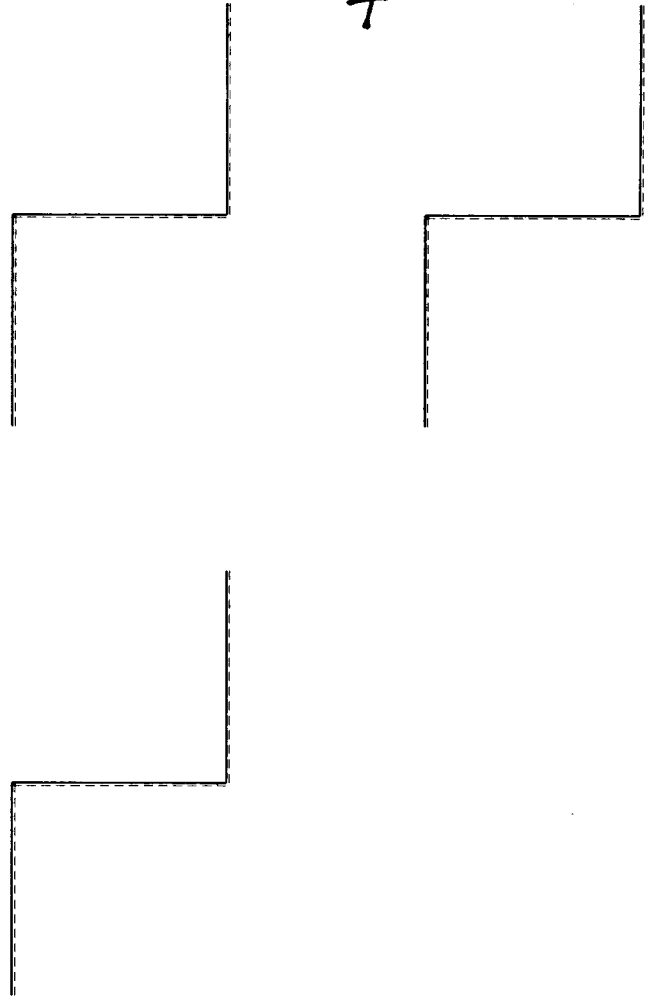
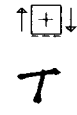
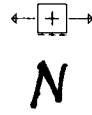
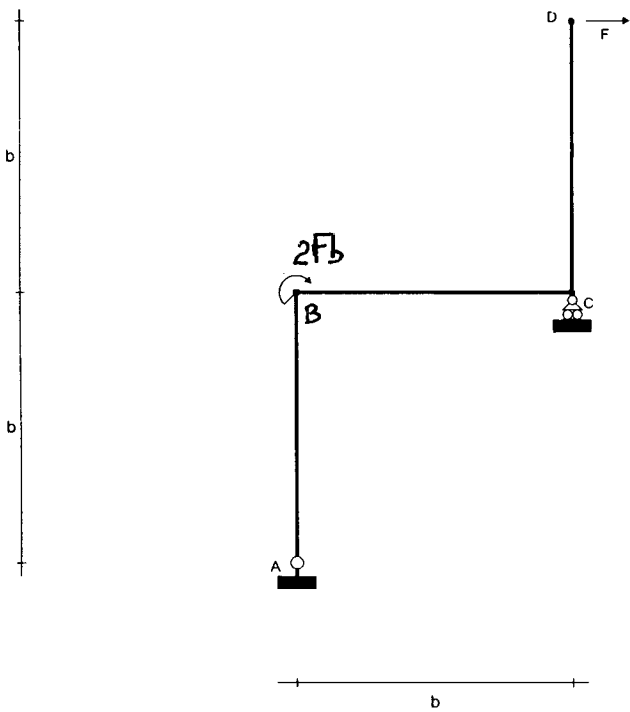


Siano $L = 8 \text{ m}$, $\alpha = 0,8$ e $\beta = 0,1$. Le due forze valgono $F = 10\text{N}$, $P = 40\text{N}$.
 Determinare il valore della distanza d tale per cui la reazione verticale V_B valga $1,2 P$ e la reazione verticale V_A per tale valore di d .

$d = \dots\dots\dots$; $V_A = \dots\dots\dots$

Esercizio n.3 (9 punti)

Per la struttura in figura, riportare le reazioni vincolari ed i diagrammi delle azioni interne M , N , T .



$V_A = \dots\dots\dots; H_A = \dots\dots\dots; V_C = \dots\dots\dots$

$N_{AB} =$

$T_{AB} =$

$M_{AB} =$

$N_{BC} =$

$T_{BC} =$

$M_{BC} =$

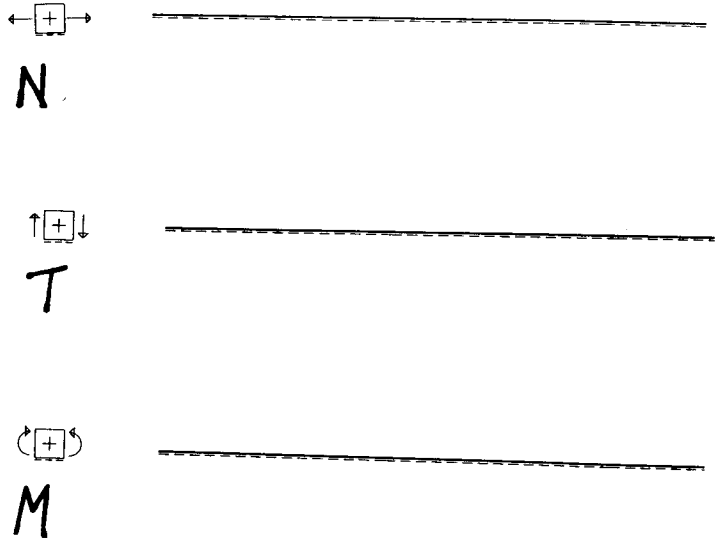
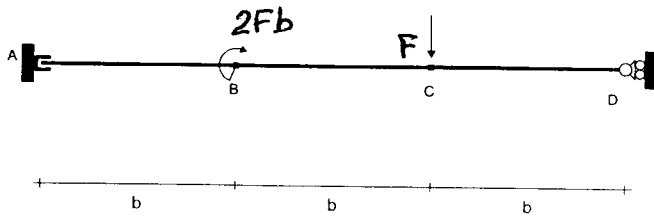
$N_{CD} =$

$T_{CD} =$

$M_{CD} =$

Esercizio n.4 (9 punti)

Per la struttura in figura, riportare le reazioni vincolari ed i diagrammi delle azioni interne M, N, T.

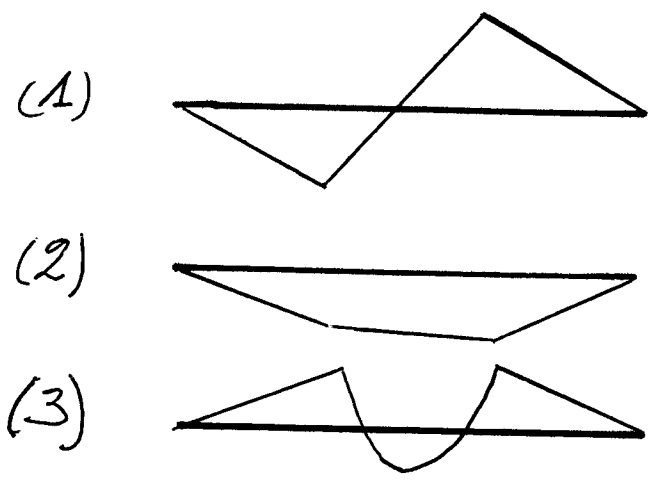
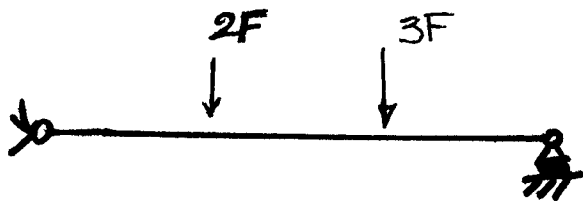


$N_{AB} =$ $N_{BC} =$ $N_{CD} =$
 $T_{AB} =$ $T_{BC} =$ $T_{CD} =$
 $M_{AB} =$ $M_{BC} =$ $M_{CD} =$

$V_A = \dots\dots\dots; M_A = \dots\dots\dots; H_D = \dots\dots\dots$

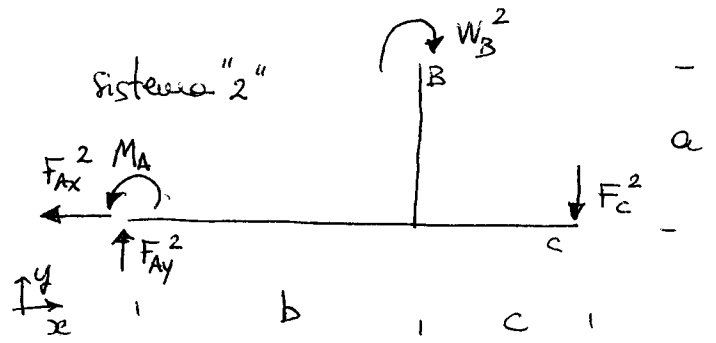
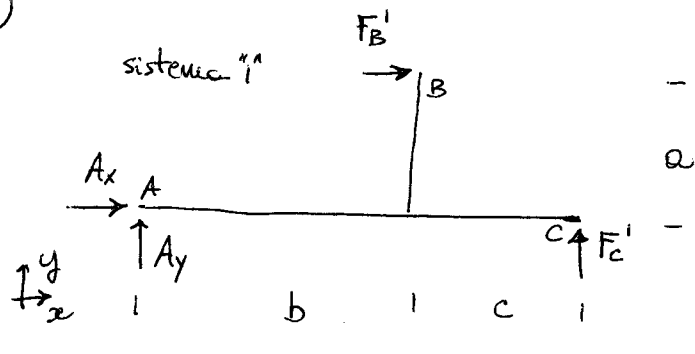
Esercizio n.5 (3 punti)

Si consideri la struttura data in figura e si indichi qual'è l'esatta distribuzione del momento flettente.



(1) (2) (3)

①



Per equivalenza statica

$$\vec{R}^1 = \vec{R}^2 \quad [0] \quad \vec{R}^1 = R_x^1 \vec{i} + R_y^1 \vec{j} + R_z^1 \vec{k}; \quad \vec{R}^2 = R_x^2 \vec{i} + R_y^2 \vec{j} + R_z^2 \vec{k}$$

$$\vec{M}_{(o)}^1 = \vec{M}_{(o)}^2 \quad [0 \cdot 0] \quad \vec{M}_{(o)}^1 = M_{x(o)}^1 \vec{i} + M_{y(o)}^1 \vec{j} + M_{z(o)}^1 \vec{k}; \quad \vec{M}_{(o)}^2 = M_{x(o)}^2 \vec{i} + M_{y(o)}^2 \vec{j} + M_{z(o)}^2 \vec{k}$$

Dalla [0] segue:

$$R_x^1 = R_x^2$$

$$R_y^1 = R_y^2$$

$$(R_z^1 = R_z^2) \leftarrow \text{automaticamente soddisfatta perche' il sistema di forze e' fisso.}$$

Dalla [0·0] segue:

$$\left. \begin{aligned} (M_{x(o)}^1 = M_{x(o)}^2) \\ (M_{y(o)}^1 = M_{y(o)}^2) \\ M_{z(o)}^1 = M_{z(o)}^2 \end{aligned} \right\} \text{ automaticamente soddisfatte perche' il sistema di forze e' fisso.}$$

Si ha così:

$$\rightarrow R_x^1 = A_x + F_B^1 = -F_{Ax}^2 = \rightarrow R_x^2$$

$$\uparrow R_y^1 = A_y + F_c^1 = F_{Ay}^2 - F_c^2 = \uparrow R_y^2$$

$$\curvearrowright M_{z(A)}^1 = (b+c)F_c^1 - a \cdot F_B^1 = M_A - W_B^2 - (b+c)F_c^2 = \curvearrowright M_{z(A)}^2$$

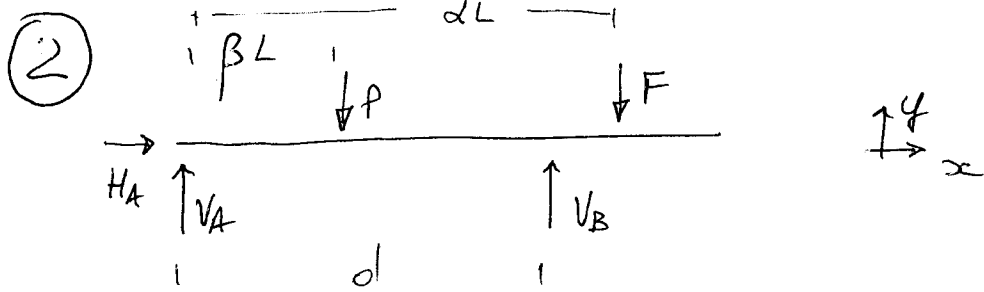
Risolvendo rispetto alle incognite A_x, A_y, M_A si trova:

$$A_x = -F_{Ax}^2 - F_B^1$$

$$A_y = +F_{Ay}^2 - F_c^1 - F_c^2$$

$$M_A = W_B^2 + (b+c)(F_c^1 + F_c^2) - aF_B^1$$

①



Per equilibrio

$$\rightarrow R_x = 0 \quad H_A = 0$$

$$\uparrow R_y = 0 \quad V_A + V_B - P - F = 0$$

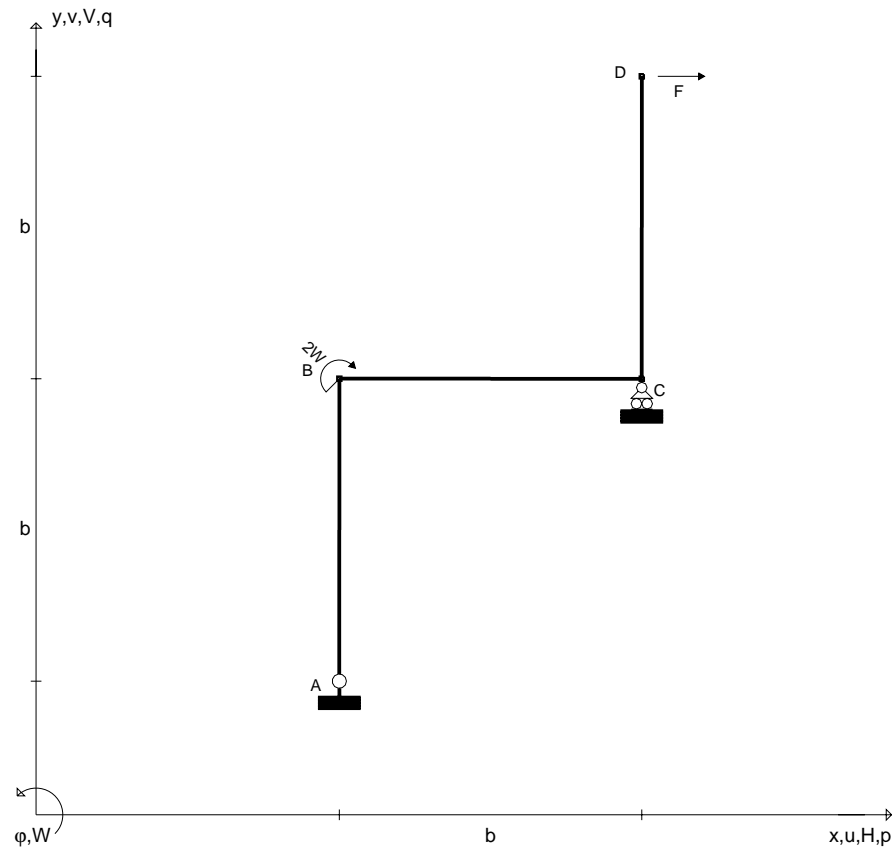
$$\curvearrowright M_{z(A)} = 0 \quad -P\beta L + V_B \cdot d - F \cdot \alpha L = 0$$

Se $F = \gamma P$ si ha:

$$V_B = \frac{P\beta L + \gamma P \cdot \alpha L}{d} = P \frac{(\beta + \alpha\gamma)L}{d}$$

$$\text{ma } V_B = \delta \cdot P \quad \Rightarrow \quad \delta \cdot P = \frac{(\beta + \alpha\gamma)L}{d} P \quad \Rightarrow \quad d = \frac{(\beta + \alpha\gamma)L}{\delta}$$

$$V_A = P + F - V_B = P + \gamma P - \delta \cdot P = P(1 + \gamma - \delta)$$

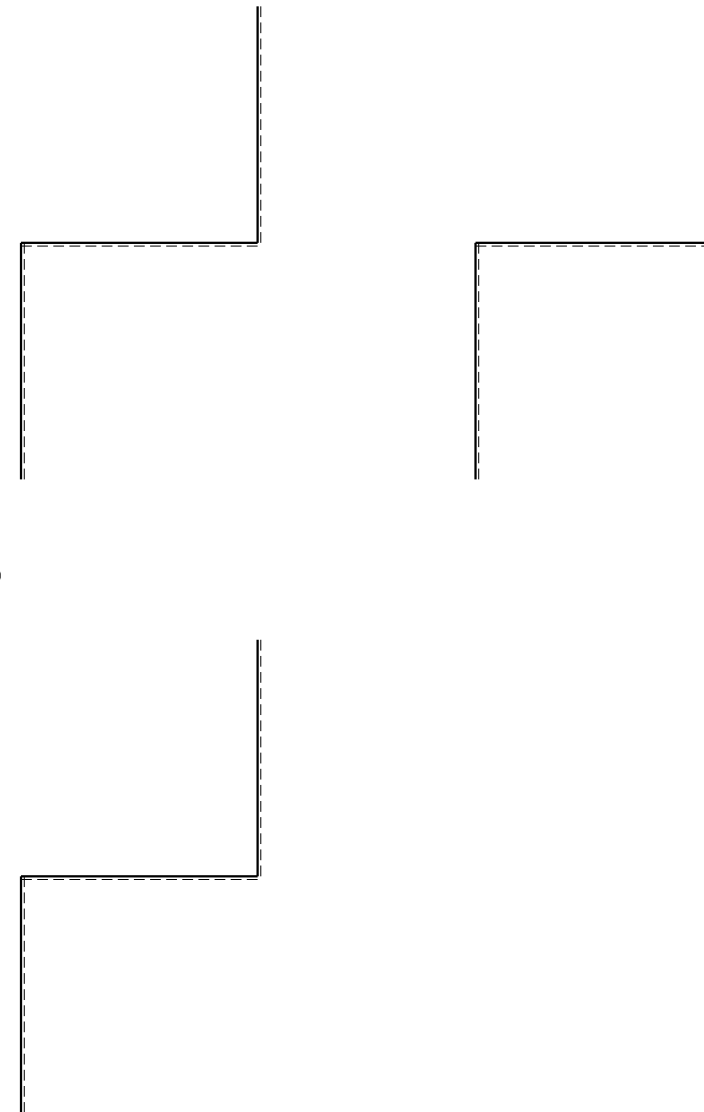
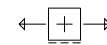


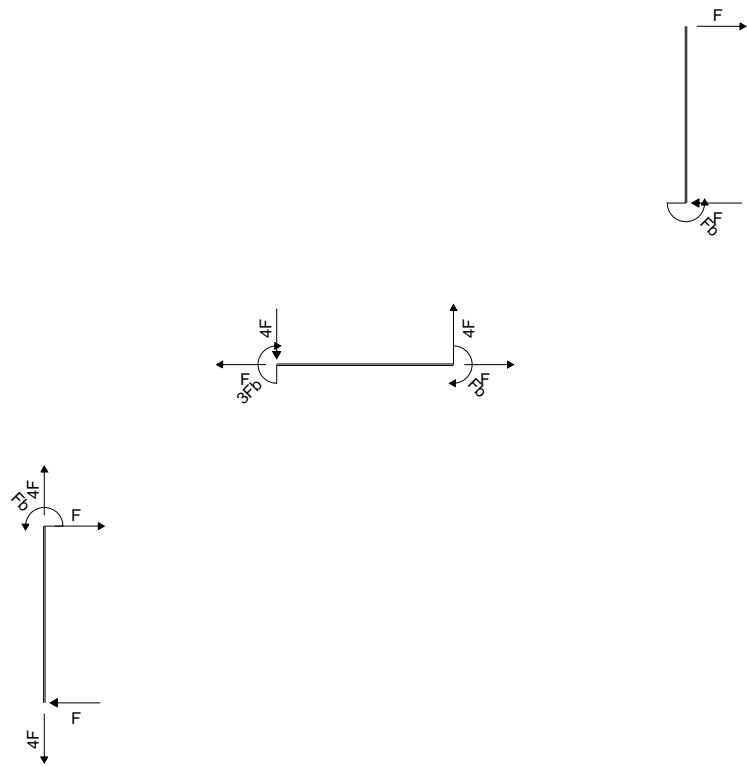
$$H_D = F \quad E_{J_{AB}} = EJ \quad E_{J_{CD}} = EJ$$

$$W_B = -2W = -2Fb \quad E_{J_{BC}} = EJ$$

Verso effettivo dei carichi riportato nel disegno.
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 Tracciare i diagrammi delle azioni interne nelle aste.
 Esprimere le funzioni delle azioni interne nelle aste.
 J_{AB} x_{AB} ϑ_{AB} riferimento locale asta AB con origine in A.

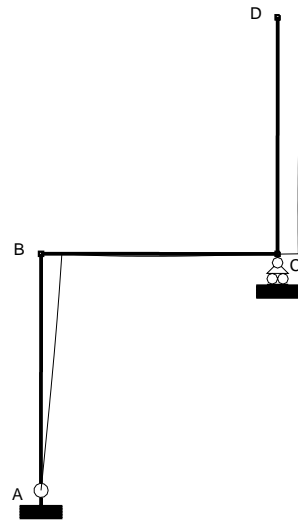
MANTENERE I RISULTATI IN FORMA FRAZIONARIA





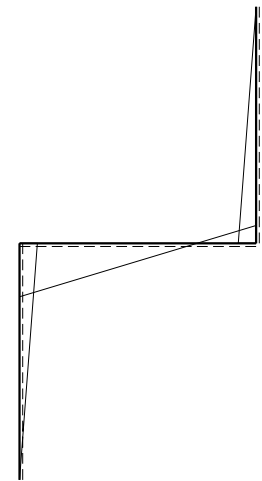
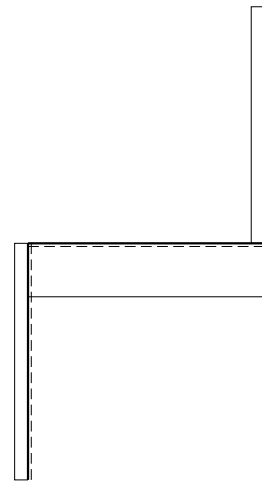
$3 Fb^3/EJ$

$14 F$



$14 F$

$13 Fb$



AZIONI INTERNE (coordinate locali)

$$N_{AB} = 4F$$

$$T_{AB} = F$$

$$M_{AB} = Fx$$

$$N_{BC} = F$$

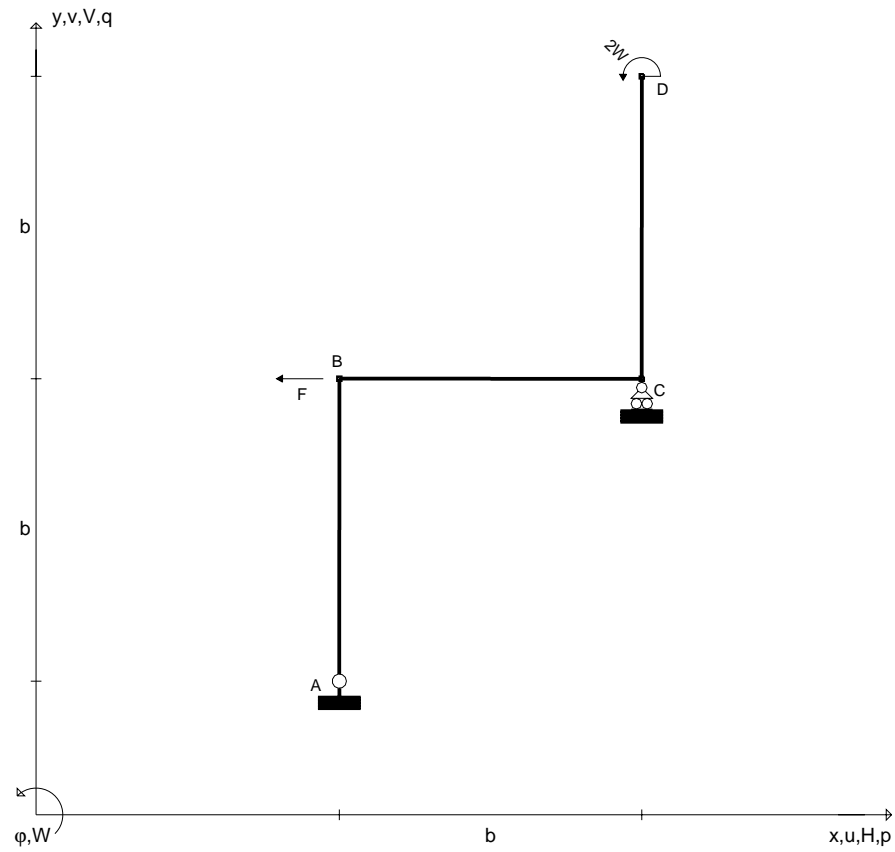
$$T_{BC} = -4F$$

$$M_{BC} = 3Fb - 4Fx$$

$$N_{CD} = 0$$

$$T_{CD} = F$$

$$M_{CD} = -Fb + Fx$$

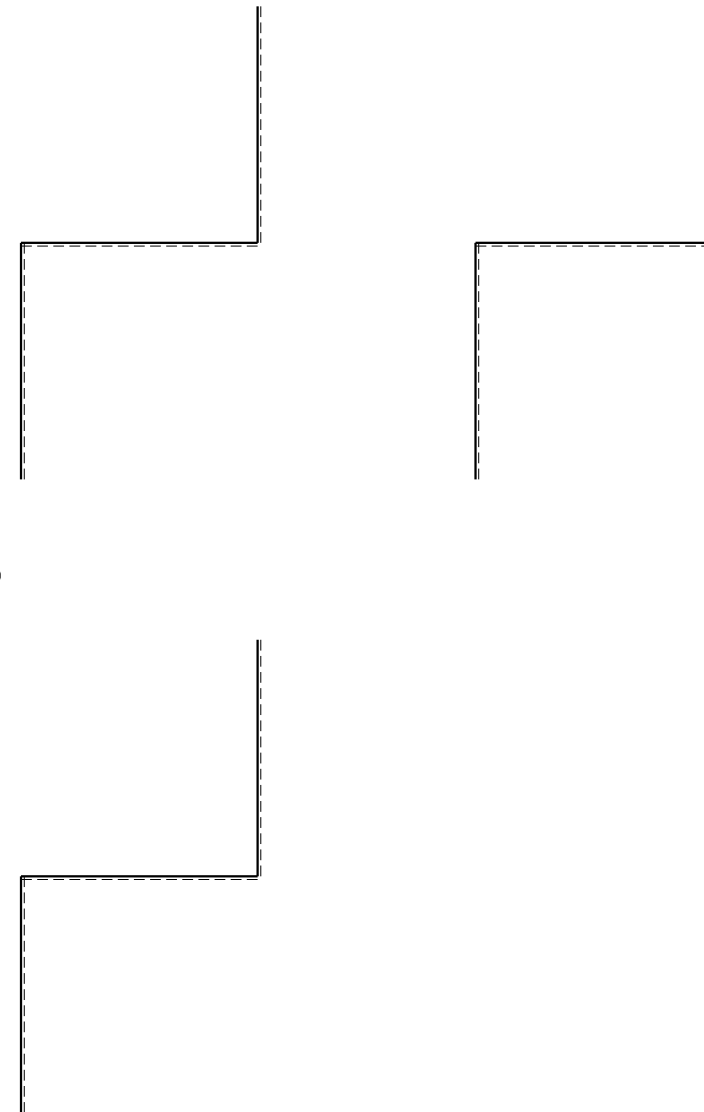
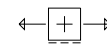


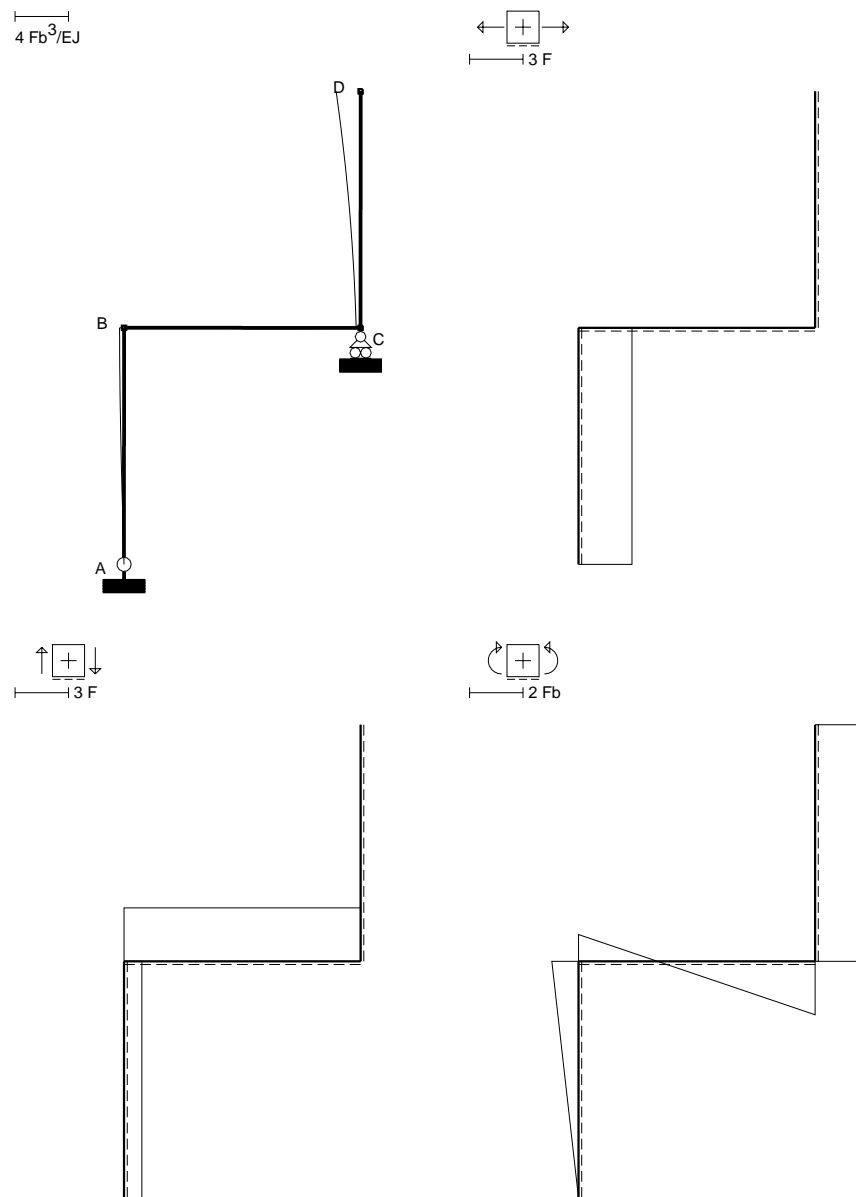
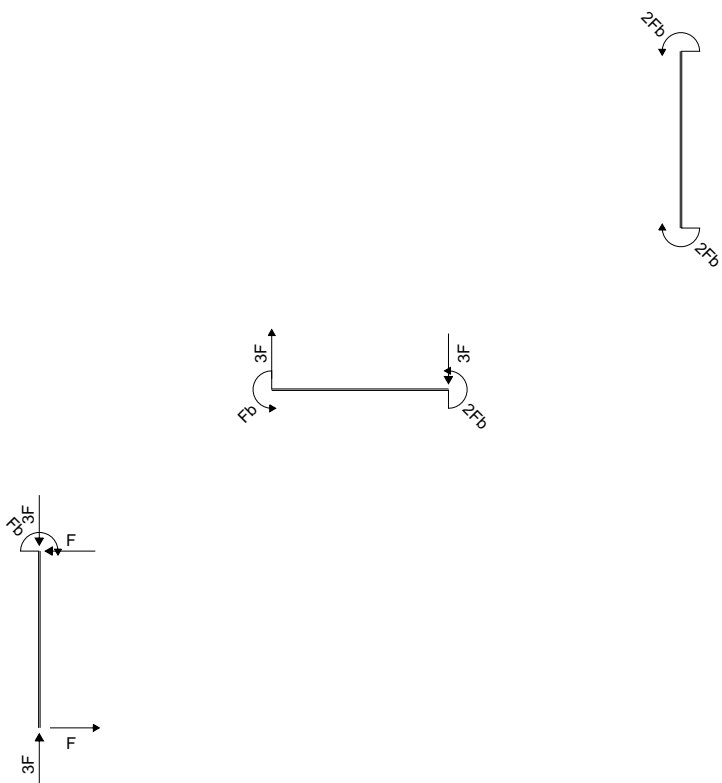
$$H_B = -F \qquad EJ_{AB} = EJ \qquad EJ_{CD} = EJ$$

$$W_D = 2W = 2Fb \qquad EJ_{BC} = EJ$$

Verso effettivo dei carichi riportato nel disegno.
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 J_{AB} x_{AB} ϑ_{AB} riferimento locale asta AB con origine in A.

MANTENERE I RISULTATI IN FORMA FRAZIONARIA





AZIONI INTERNE (coordinate locali)

$$N_{AB} = -3F$$

$$T_{AB} = -F$$

$$M_{AB} = -Fx$$

$$N_{BC} = 0$$

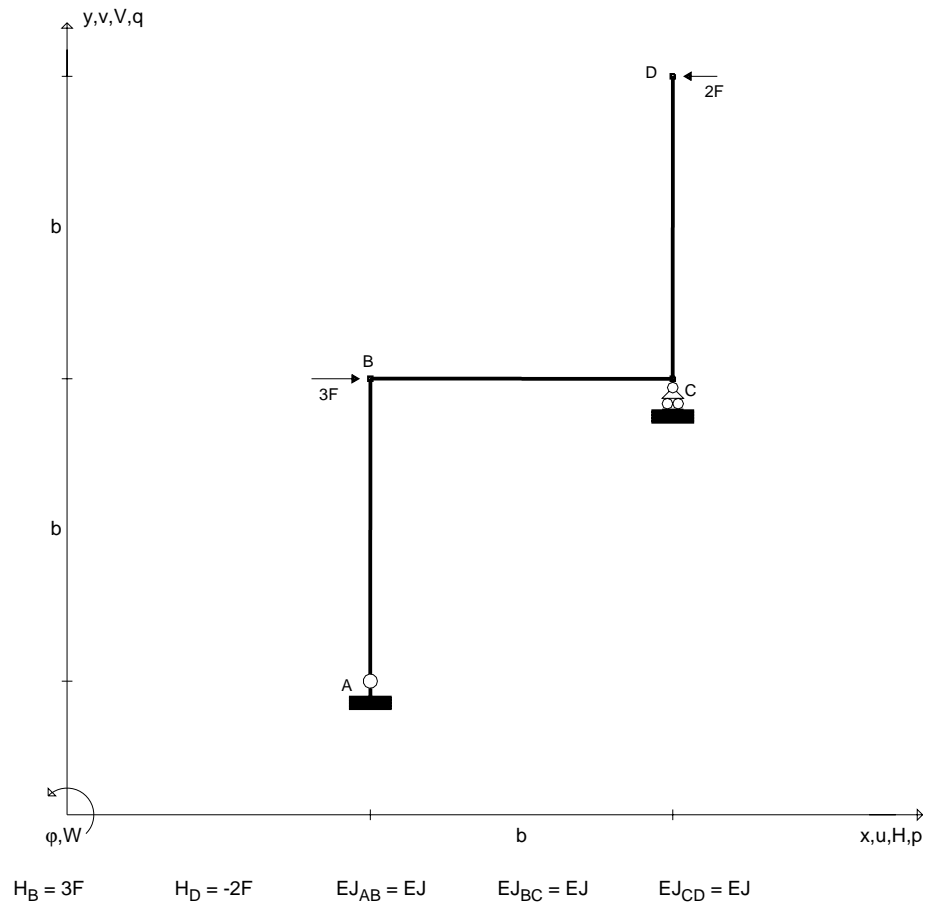
$$T_{BC} = 3F$$

$$M_{BC} = -Fb + 3Fx$$

$$N_{CD} = 0$$

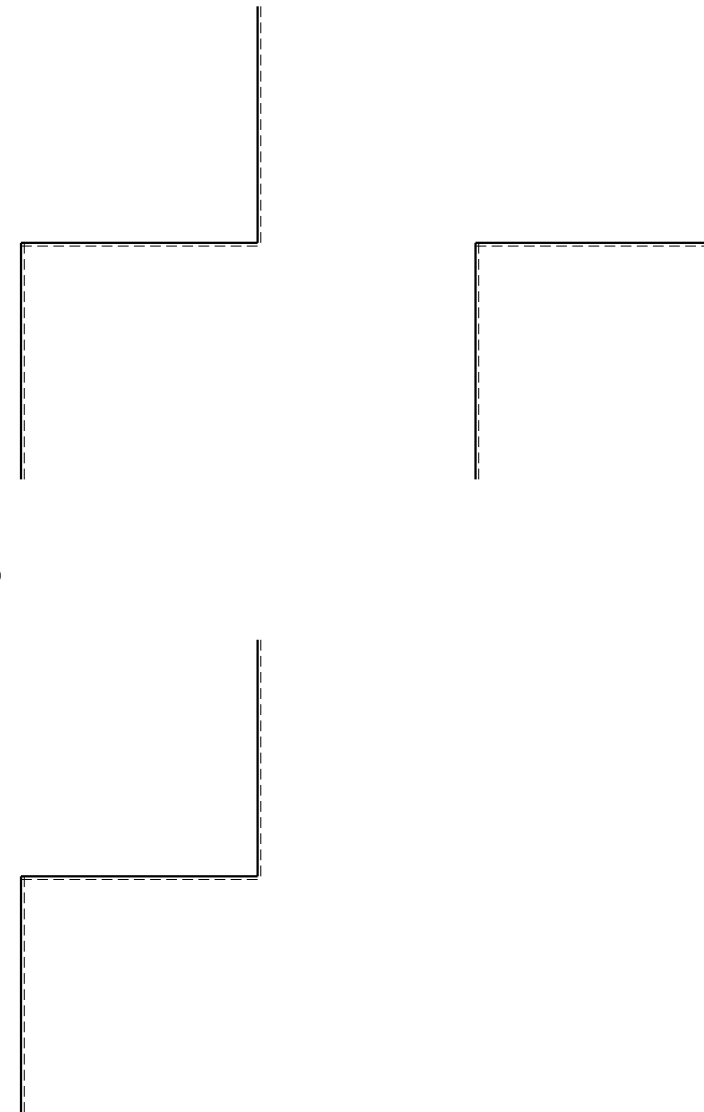
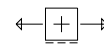
$$T_{CD} = 0$$

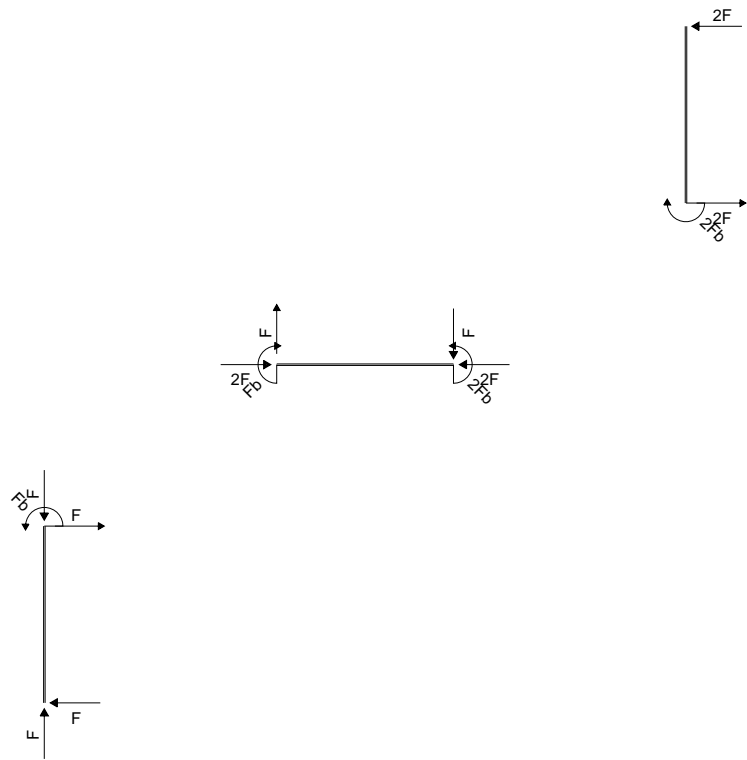
$$M_{CD} = 2Fb$$



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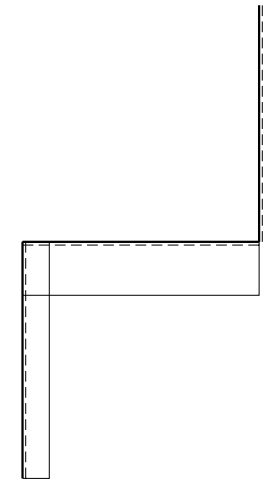
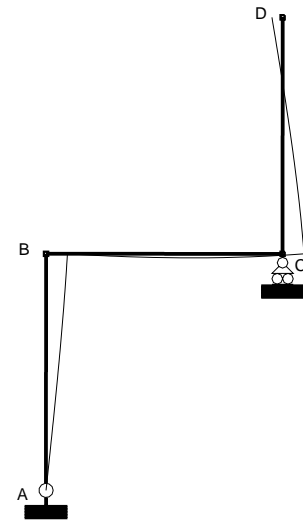
MANTENERE I RISULTATI IN FORMA FRAZIONARIA





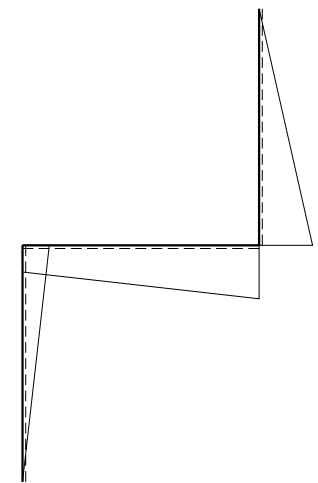
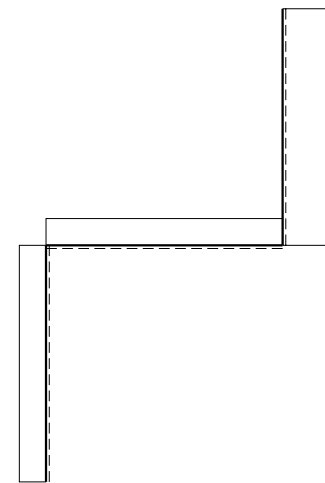
$2.5 Fb^3/EJ$

$12 F$



$12 F$

$12 Fb$



AZIONI INTERNE (coordinate locali)

$$N_{AB} = -F$$

$$T_{AB} = F$$

$$M_{AB} = Fx$$

$$N_{BC} = -2F$$

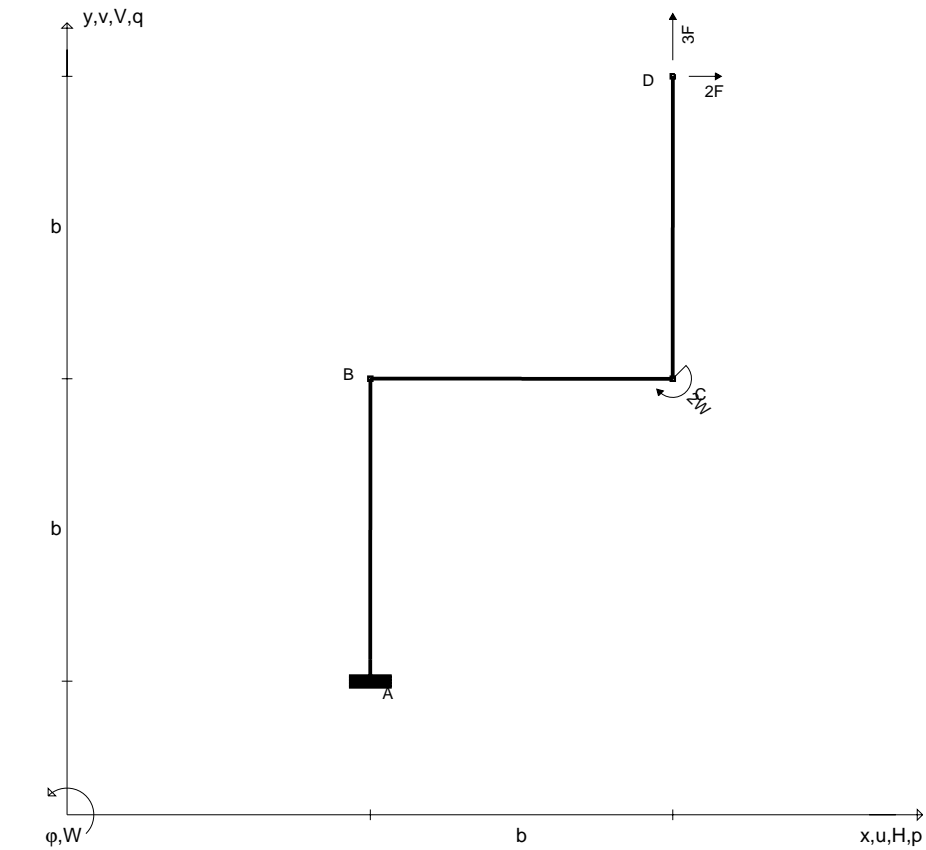
$$T_{BC} = F$$

$$M_{BC} = Fb + Fx$$

$$N_{CD} = 0$$

$$T_{CD} = -2F$$

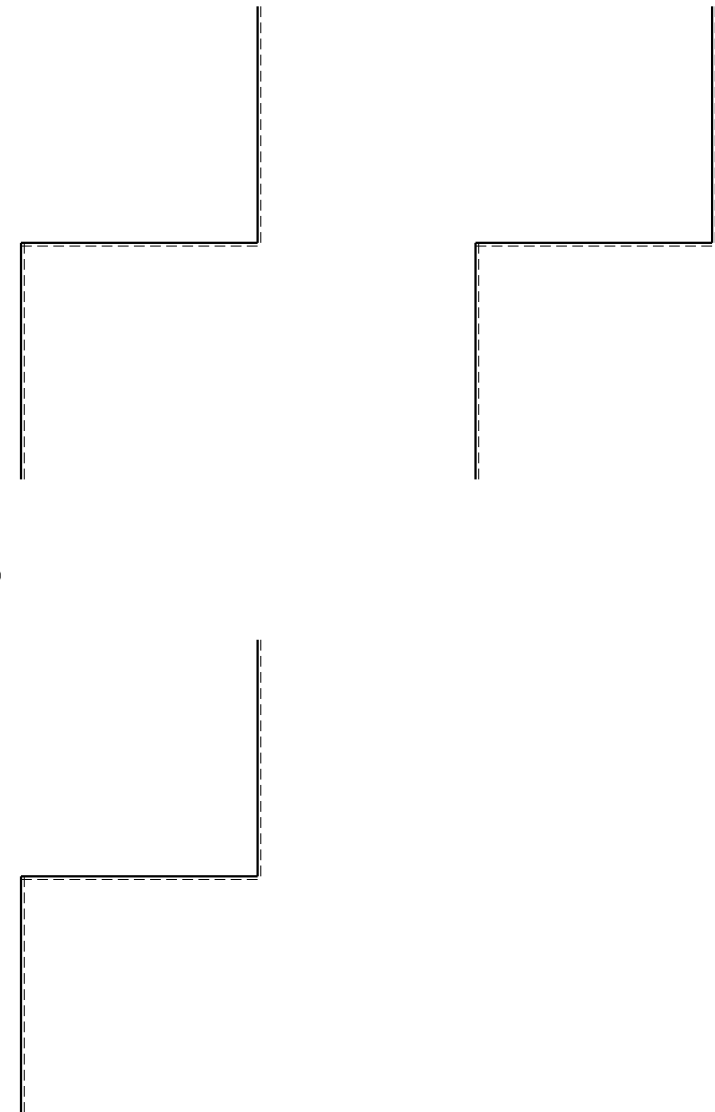
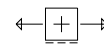
$$M_{CD} = 2Fb - 2Fx$$

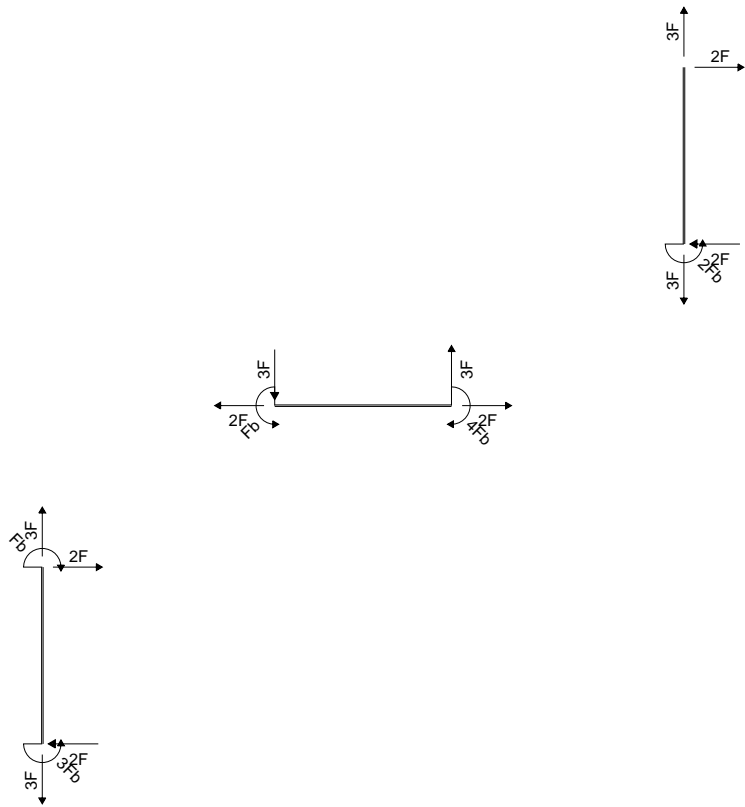


$V_D = 3F$	$W_C = -2W = -2Fb$	$EJ_{BC} = EJ$
$H_D = 2F$	$EJ_{AB} = EJ$	$EJ_{CD} = EJ$

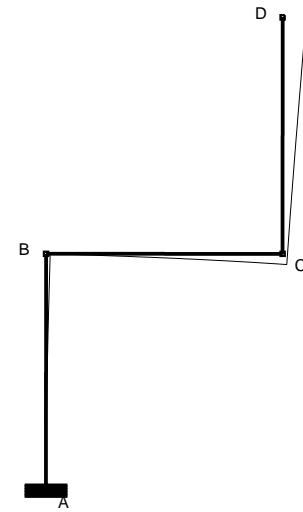
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MANTENERE I RISULTATI IN FORMA FRAZIONARIA

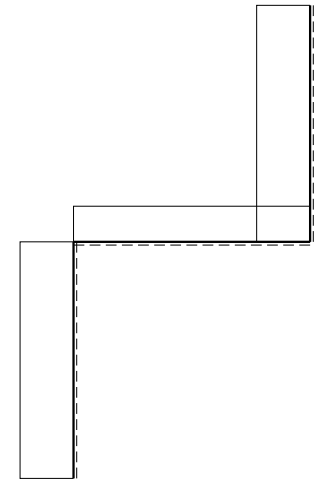




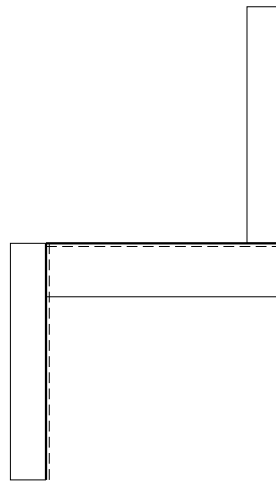
$15 Fb^3/EJ$



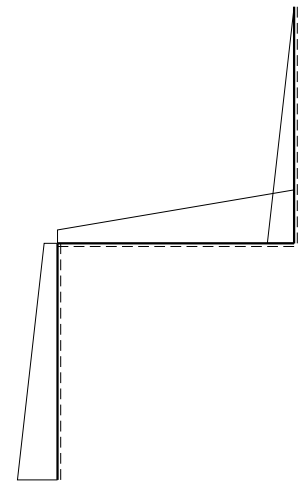
$13 F$



$13 F$



$14 Fb$



AZIONI INTERNE (coordinate locali)

$$N_{AB} = 3F$$

$$T_{AB} = 2F$$

$$M_{AB} = -3Fb + 2Fx$$

$$N_{BC} = 2F$$

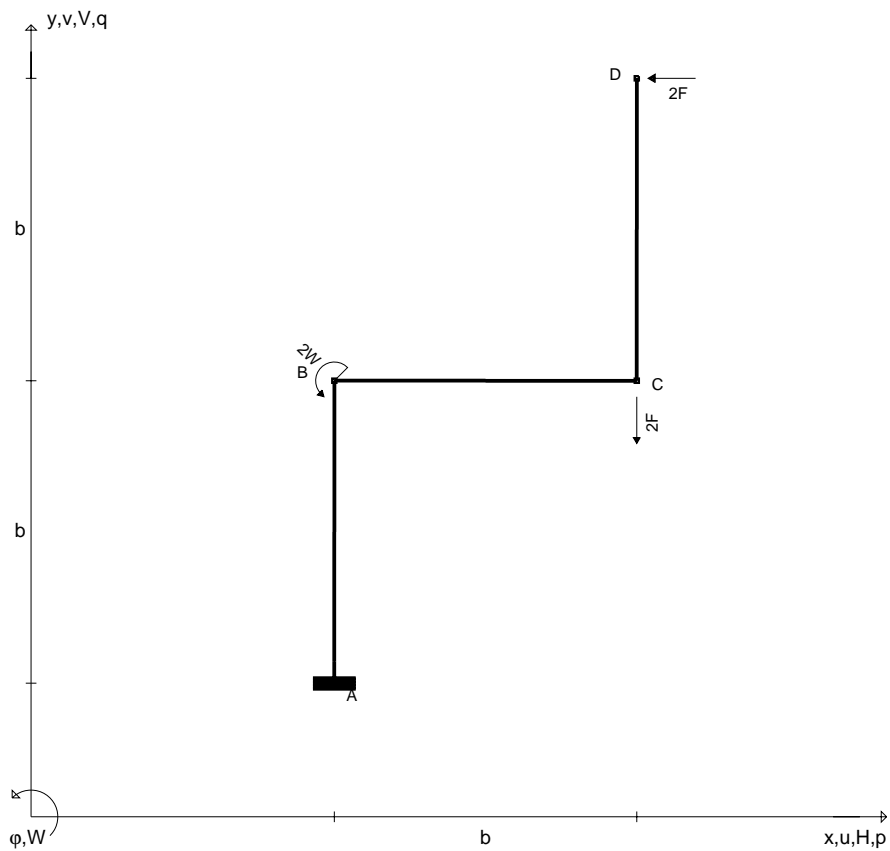
$$T_{BC} = -3F$$

$$M_{BC} = -Fb - 3Fx$$

$$N_{CD} = 3F$$

$$T_{CD} = 2F$$

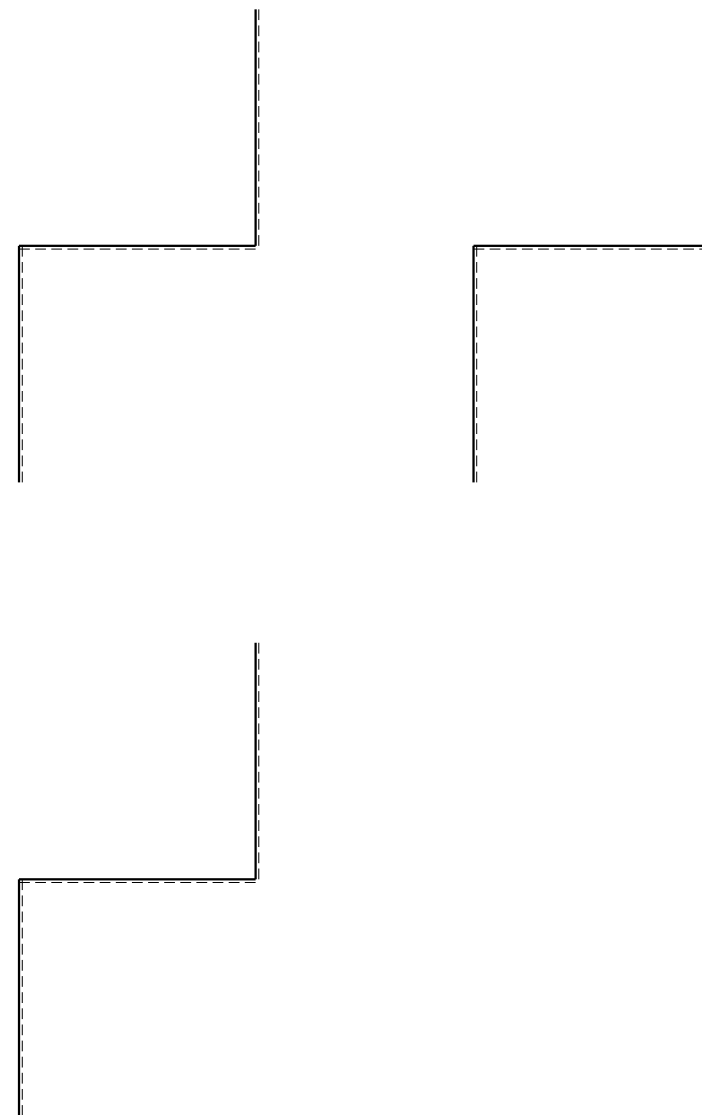
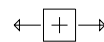
$$M_{CD} = -2Fb + 2Fx$$

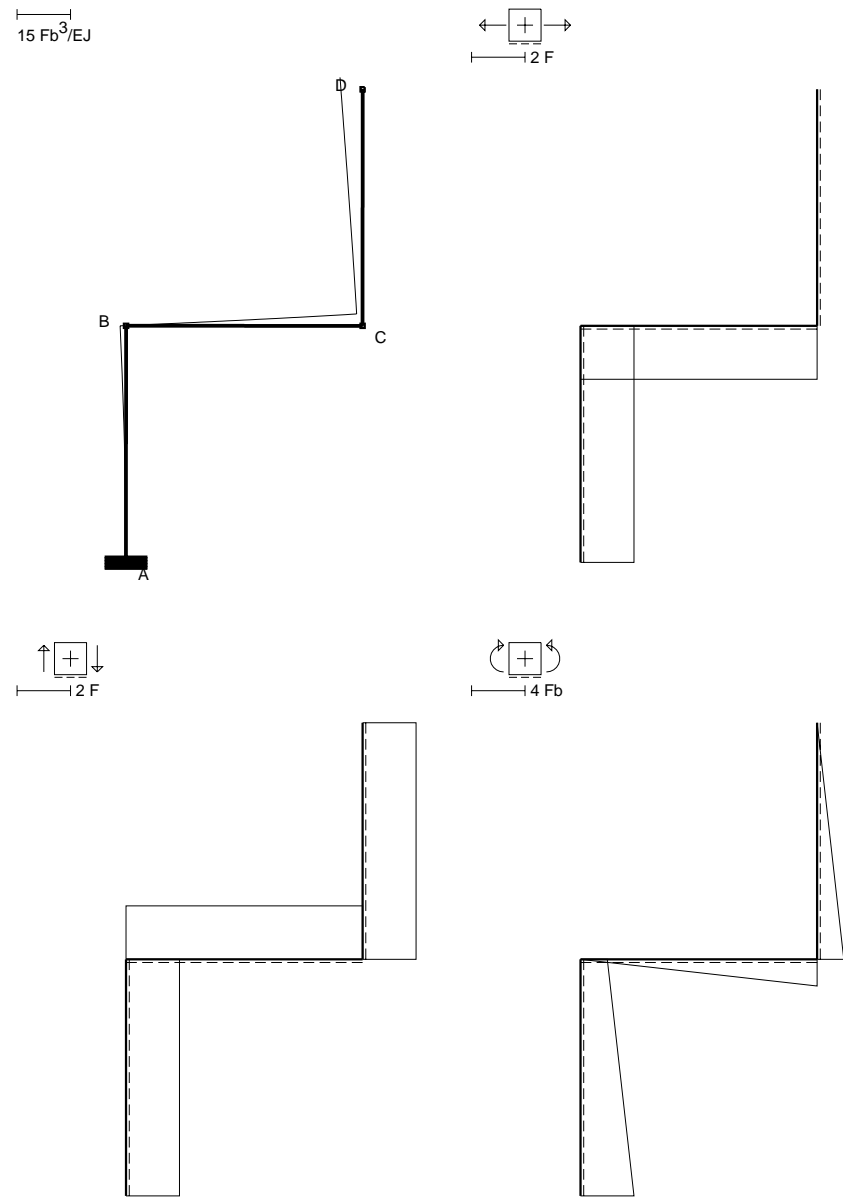
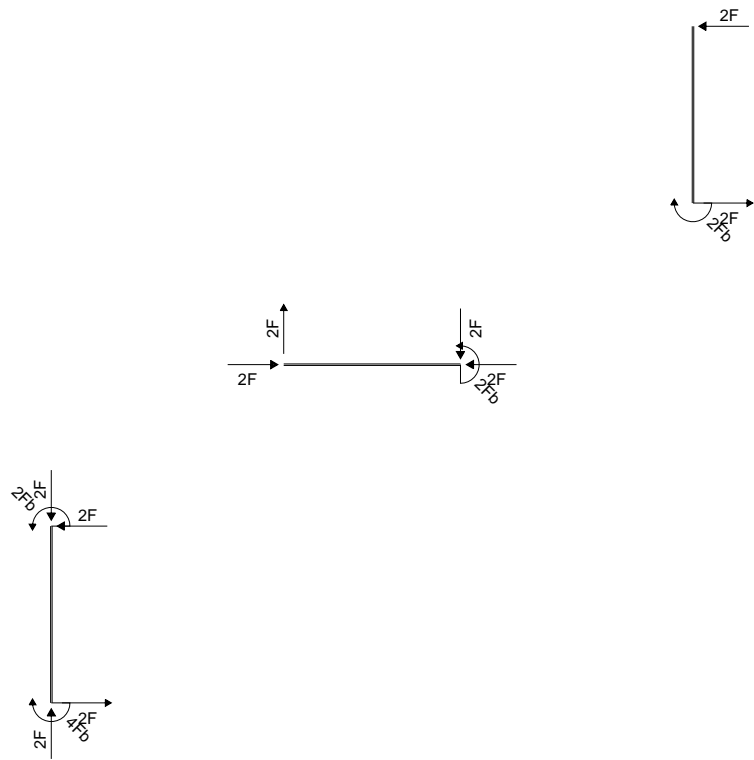


$H_D = -2F$	$W_B = 2W = 2Fb$	$EJ_{BC} = EJ$
$V_C = -2F$	$EJ_{AB} = EJ$	$EJ_{CD} = EJ$

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 J_{AB} x_{AB} ϑ_{AB} riferimento locale asta AB con origine in A.

MANTENERE I RISULTATI IN FORMA FRAZIONARIA





AZIONI INTERNE (coordinate locali)

$$N_{AB} = -2F$$

$$T_{AB} = -2F$$

$$M_{AB} = 4Fb - 2Fx$$

$$N_{BC} = -2F$$

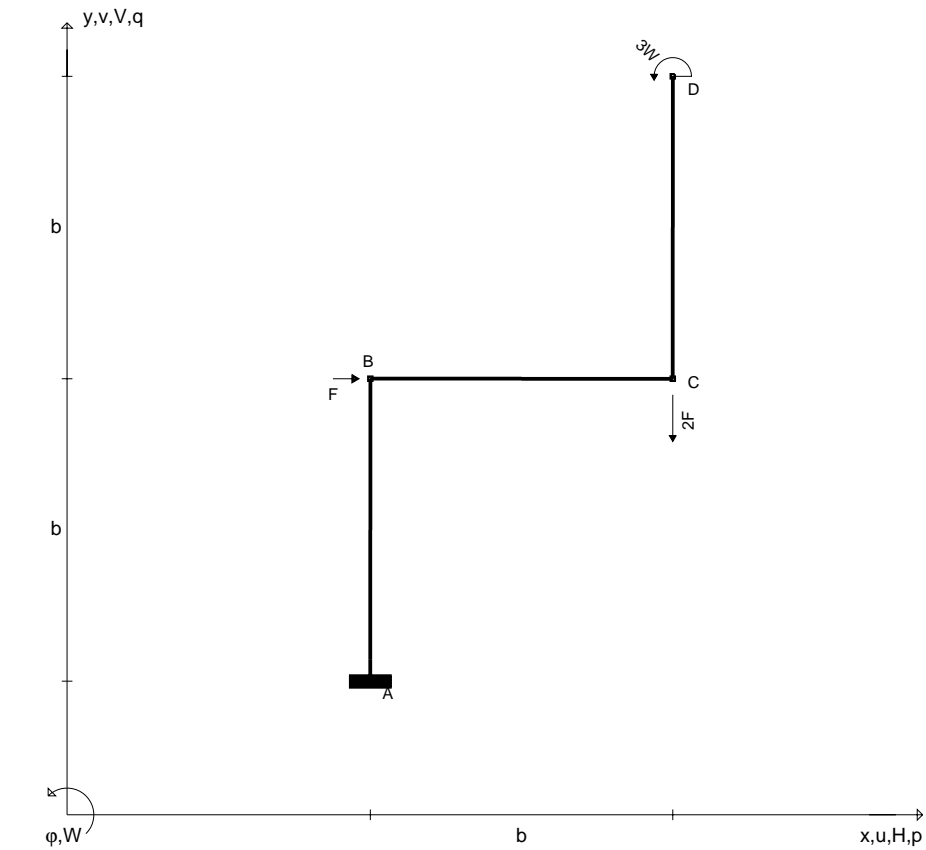
$$T_{BC} = 2F$$

$$M_{BC} = 2Fx$$

$$N_{CD} = 0$$

$$T_{CD} = -2F$$

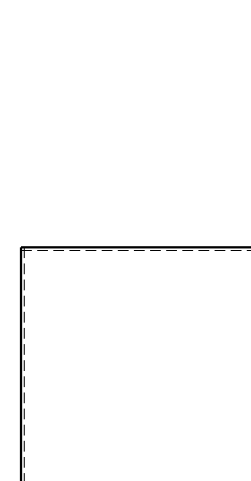
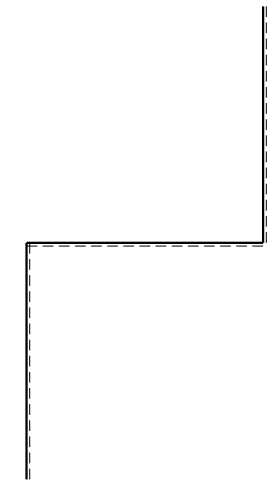
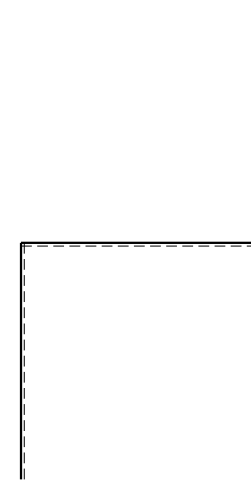
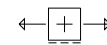
$$M_{CD} = 2Fb - 2Fx$$

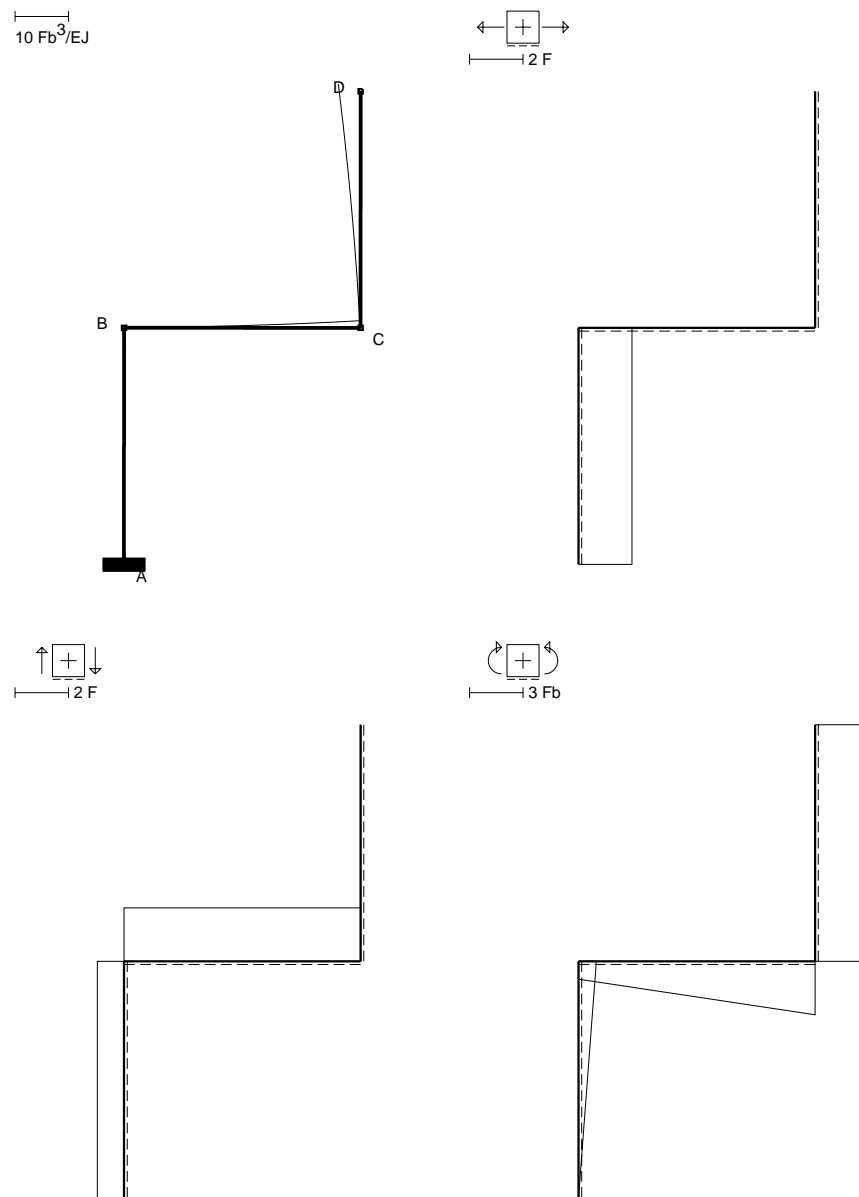
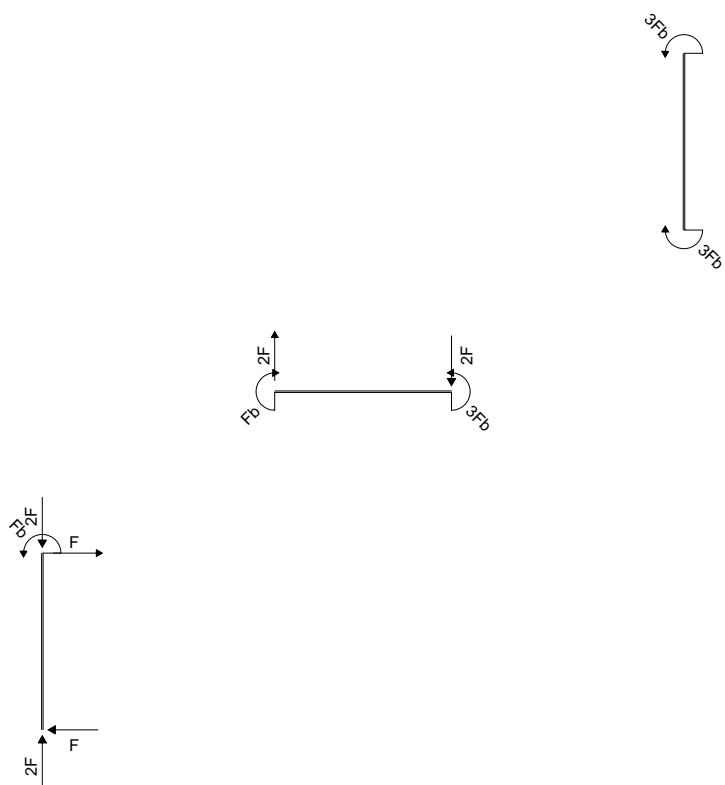


$V_C = -2F$	$W_D = 3W = 3Fb$	$EJ_{BC} = EJ$
$H_B = F$	$EJ_{AB} = EJ$	$EJ_{CD} = EJ$

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MANTENERE I RISULTATI IN FORMA FRAZIONARIA





AZIONI INTERNE (coordinate locali)

$$N_{AB} = -2F$$

$$T_{AB} = F$$

$$M_{AB} = Fx$$

$$N_{BC} = 0$$

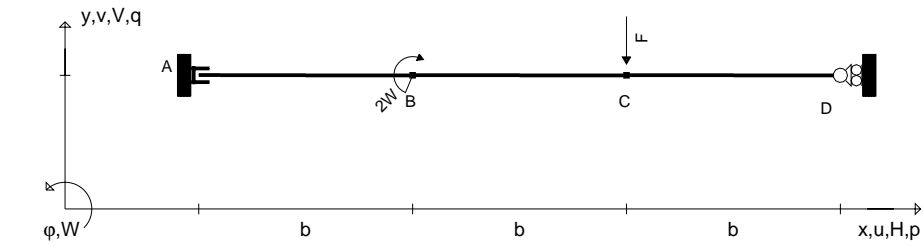
$$T_{BC} = 2F$$

$$M_{BC} = Fb + 2Fx$$

$$N_{CD} = 0$$

$$T_{CD} = 0$$

$$M_{CD} = 3Fb$$



$$V_C = -F$$

$$W_B = -2W = -2Fb$$

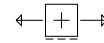
$$EJ_{AB} = EJ$$

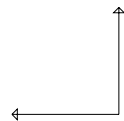
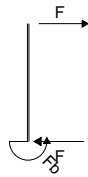
$$EJ_{BC} = EJ$$

$$EJ_{CD} = EJ$$

Verso effettivo dei carichi riportato nel disegno.
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MANTENERE I RISULTATI IN FORMA FRAZIONARIA

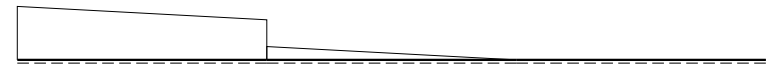




$20 \frac{Fb^3}{EJ}$



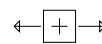
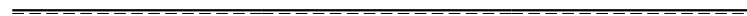
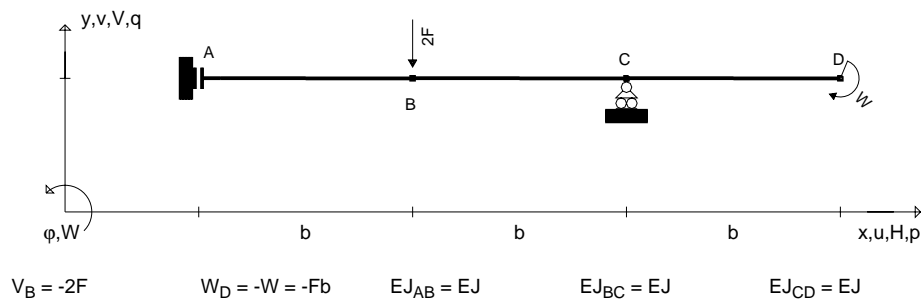
$1 F$



$4 Fb$

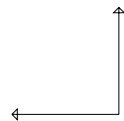
AZIONI INTERNE (coordinate locali)

$N_{AB} = 0$	$N_{BC} = 0$	$N_{CD} = 0$
$T_{AB} = F$	$T_{BC} = F$	$T_{CD} = 0$
$M_{AB} = -4Fb + Fx$	$M_{BC} = -Fb + Fx$	$M_{CD} = 0$



Verso effettivo dei carichi riportato nel disegno.
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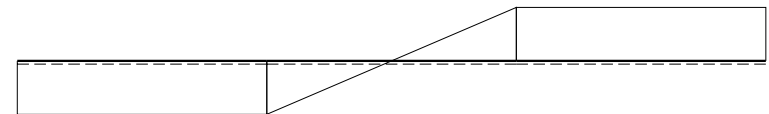
MANTENERE I RISULTATI IN FORMA FRAZIONARIA



$4 F_b^3 / EJ$



$\uparrow + \downarrow$
 $12 F$



$\curvearrowright + \curvearrowleft$
 $1 F_b$

AZIONI INTERNE (coordinate locali)

$$N_{AB} = 0$$

$$T_{AB} = 0$$

$$M_{AB} = Fb$$

$$N_{BC} = 0$$

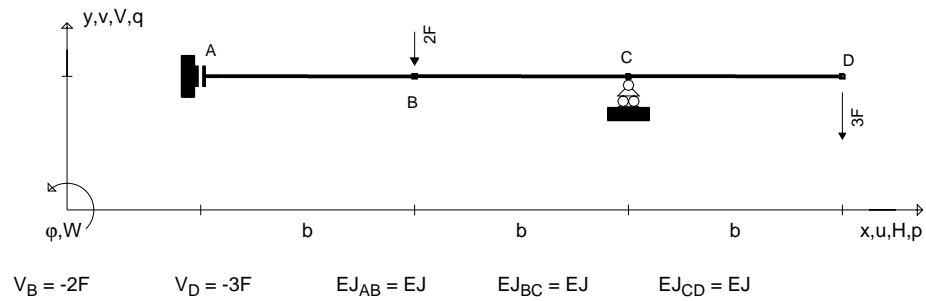
$$T_{BC} = -2F$$

$$M_{BC} = Fb - 2Fx$$

$$N_{CD} = 0$$

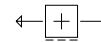
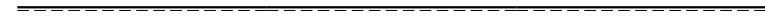
$$T_{CD} = 0$$

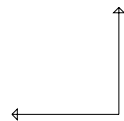
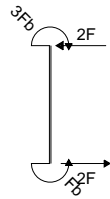
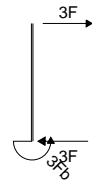
$$M_{CD} = -Fb$$



Verso effettivo dei carichi riportato nel disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi delle azioni interne nelle aste.
 Esprimere le funzioni delle azioni interne nelle aste.
 J_{AB} x_{AB} ϑ_{AB} riferimento locale asta AB con origine in A.

MANTENERE I RISULTATI IN FORMA FRAZIONARIA

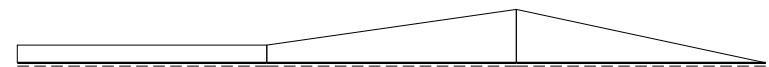




$10 Fb^3/EJ$



$13 F$



$13 Fb$

AZIONI INTERNE (coordinate locali)

$$N_{AB} = 0$$

$$T_{AB} = 0$$

$$M_{AB} = -Fb$$

$$N_{BC} = 0$$

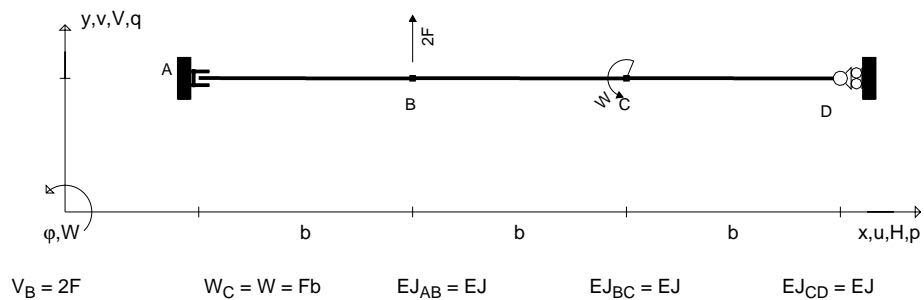
$$T_{BC} = -2F$$

$$M_{BC} = -Fb - 2Fx$$

$$N_{CD} = 0$$

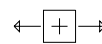
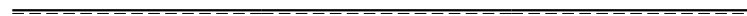
$$T_{CD} = 3F$$

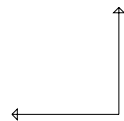
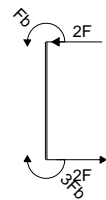
$$M_{CD} = -3Fb + 3Fx$$



Verso effettivo dei carichi riportato nel disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi delle azioni interne nelle aste.
 Esprimere le funzioni delle azioni interne nelle aste.
 J_{AB} x_{AB} ϑ_{AB} riferimento locale asta AB con origine in A.

MANTENERE I RISULTATI IN FORMA FRAZIONARIA





$15 Fb^3/EJ$



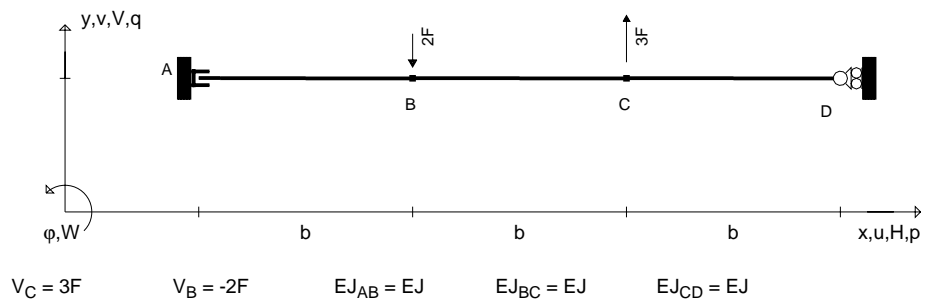
$12 F$



$13 Fb$

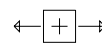
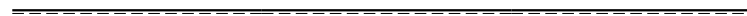
AZIONI INTERNE (coordinate locali)

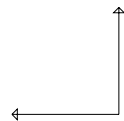
$N_{AB} = 0$	$N_{BC} = 0$	$N_{CD} = 0$
$T_{AB} = -2F$	$T_{BC} = 0$	$T_{CD} = 0$
$M_{AB} = 3Fb - 2Fx$	$M_{BC} = Fb$	$M_{CD} = 0$



Verso effettivo dei carichi riportato nel disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi delle azioni interne nelle aste.
 Esprimere le funzioni delle azioni interne nelle aste.
 J_{AB} x_{AB} ϑ_{AB} riferimento locale asta AB con origine in A.

MANTENERE I RISULTATI IN FORMA FRAZIONARIA

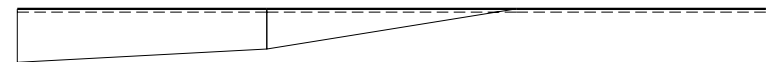




$25 Fb^3/EJ$



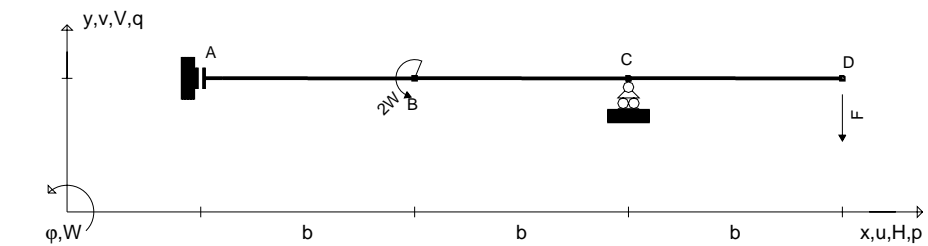
$13 F$



$4 Fb$

AZIONI INTERNE (coordinate locali)

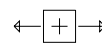
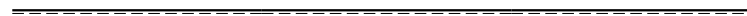
$N_{AB} = 0$	$N_{BC} = 0$	$N_{CD} = 0$
$T_{AB} = -F$	$T_{BC} = -3F$	$T_{CD} = 0$
$M_{AB} = 4Fb - Fx$	$M_{BC} = 3Fb - 3Fx$	$M_{CD} = 0$



$V_D = -F$
 $W_B = 2W = 2Fb$

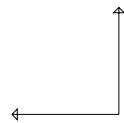
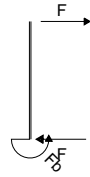
$EJ_{AB} = EJ$
 $EJ_{BC} = EJ$

$EJ_{CD} = EJ$

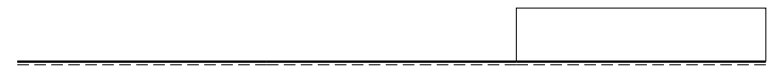


Verso effettivo dei carichi riportato nel disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi delle azioni interne nelle aste.
 Esprimere le funzioni delle azioni interne nelle aste.
 J_{AB} x_{AB} ϑ_{AB} riferimento locale asta AB con origine in A.

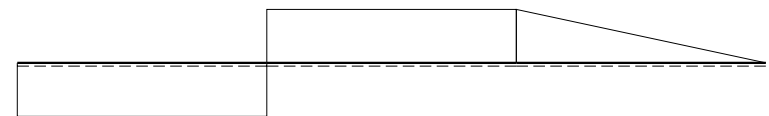
MANTENERE I RISULTATI IN FORMA FRAZIONARIA



$2.5 Fb^3/EJ$



$\uparrow + \downarrow$
 $1 F$



$\curvearrowright + \curvearrowleft$
 $1 Fb$

AZIONI INTERNE (coordinate locali)

$$N_{AB} = 0$$

$$T_{AB} = 0$$

$$M_{AB} = Fb$$

$$N_{BC} = 0$$

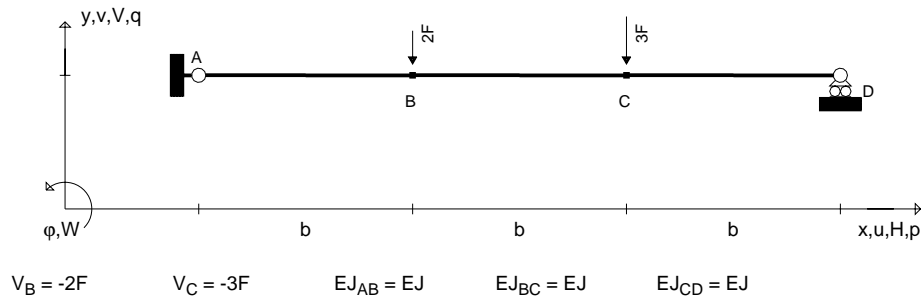
$$T_{BC} = 0$$

$$M_{BC} = -Fb$$

$$N_{CD} = 0$$

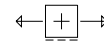
$$T_{CD} = F$$

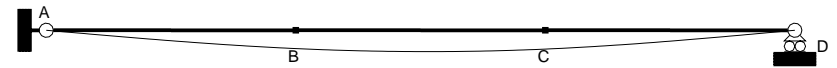
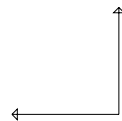
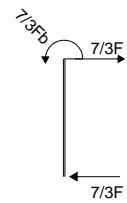
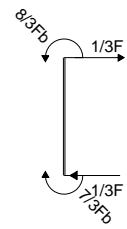
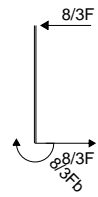
$$M_{CD} = -Fb + Fx$$



Verso effettivo dei carichi riportato nel disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi delle azioni interne nelle aste.
 Esprimere le funzioni delle azioni interne nelle aste.
 J_{AB} x_{AB} ϑ_{AB} riferimento locale asta AB con origine in A.

MANTENERE I RISULTATI IN FORMA FRAZIONARIA





$6 Fb^3/EJ$



$\uparrow + \downarrow$
 $12.5 F$



$\curvearrowright + \curvearrowleft$
 $12.5 Fb$

AZIONI INTERNE (coordinate locali)

$$N_{AB} = 0$$

$$T_{AB} = 7/3F$$

$$M_{AB} = 7/3Fx$$

$$N_{BC} = 0$$

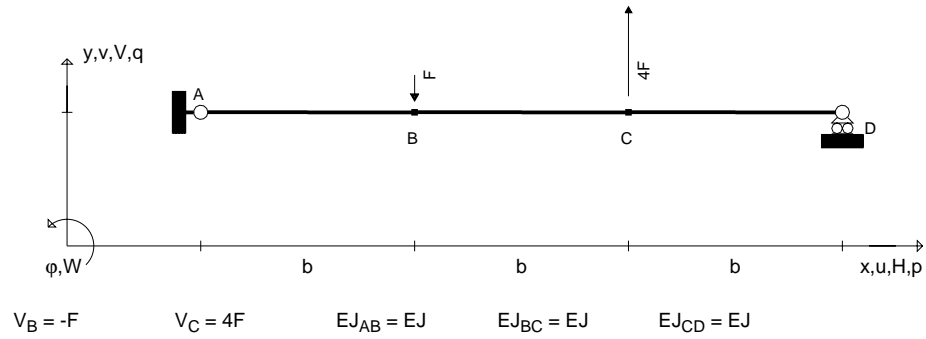
$$T_{BC} = 1/3F$$

$$M_{BC} = 7/3Fb + 1/3Fx$$

$$N_{CD} = 0$$

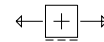
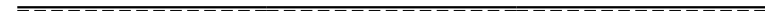
$$T_{CD} = -8/3F$$

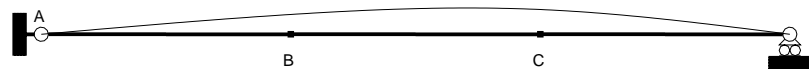
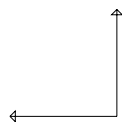
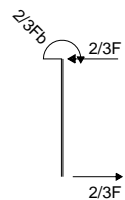
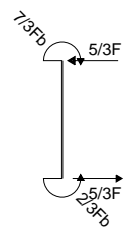
$$M_{CD} = 8/3Fb - 8/3Fx$$



Verso effettivo dei carichi riportato nel disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi delle azioni interne nelle aste.
 Esprimere le funzioni delle azioni interne nelle aste.
 J_{AB} x_{AB} ϑ_{AB} riferimento locale asta AB con origine in A.

MANTENERE I RISULTATI IN FORMA FRAZIONARIA





$3 Fb^3/EJ$



$\uparrow + \downarrow$
 $2 F$



$\curvearrowright + \curvearrowleft$
 $2 Fb$

AZIONI INTERNE (coordinate locali)

$$N_{AB} = 0$$

$$T_{AB} = -2/3F$$

$$M_{AB} = -2/3Fx$$

$$N_{BC} = 0$$

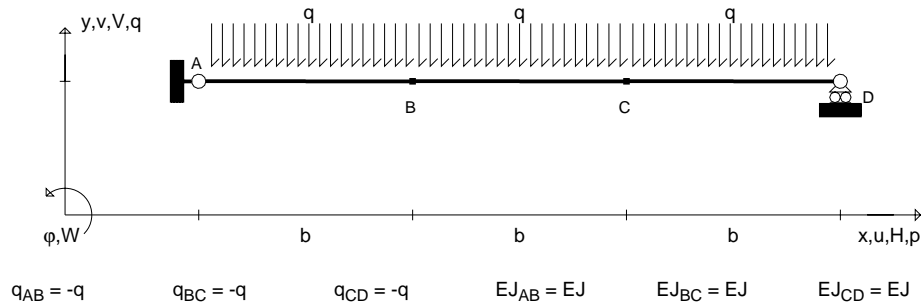
$$T_{BC} = -5/3F$$

$$M_{BC} = -2/3Fb - 5/3Fx$$

$$N_{CD} = 0$$

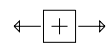
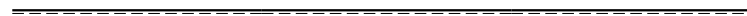
$$T_{CD} = 7/3F$$

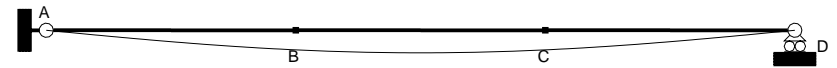
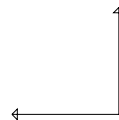
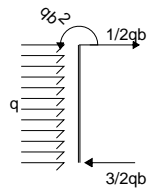
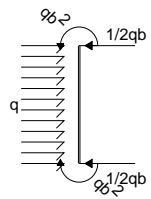
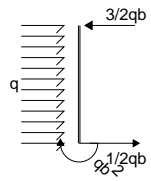
$$M_{CD} = -7/3Fb + 7/3Fx$$



Verso effettivo dei carichi riportato nel disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi delle azioni interne nelle aste.
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 x_{AB} riferimento locale asta AB con origine in A.

MANTENERE I RISULTATI IN FORMA FRAZIONARIA





$2.5 \frac{qb^4}{EJ}$



$1.5 qb$



$1 qb^2$

AZIONI INTERNE (coordinate locali)

$$N_{AB} = 0$$

$$T_{AB} = 3/2qb - qx$$

$$M_{AB} = 3/2qbx - 1/2qx^2$$

$$N_{BC} = 0$$

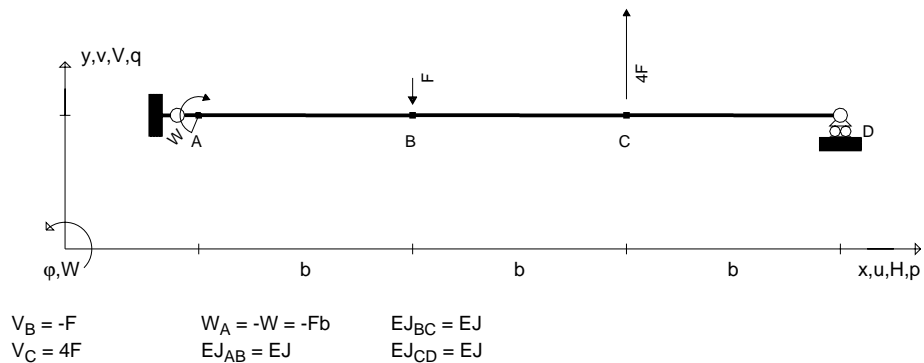
$$T_{BC} = 1/2qb - qx$$

$$M_{BC} = qb^2 + 1/2qbx - 1/2qx^2$$

$$N_{CD} = 0$$

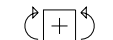
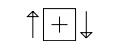
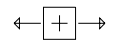
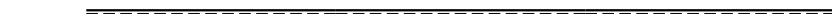
$$T_{CD} = -1/2qb - qx$$

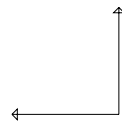
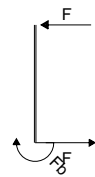
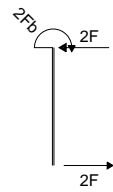
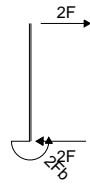
$$M_{CD} = qb^2 - 1/2qbx - 1/2qx^2$$



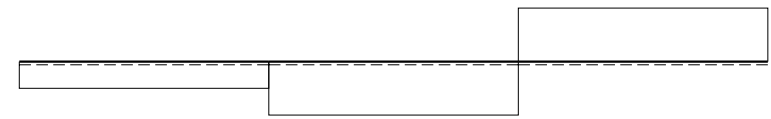
Verso effettivo dei carichi riportato nel disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi delle azioni interne nelle aste.
 Esprimere le funzioni delle azioni interne nelle aste.
 J_{AB} x_{AB} ϑ_{AB} riferimento locale asta AB con origine in A.

MANTENERE I RISULTATI IN FORMA FRAZIONARIA

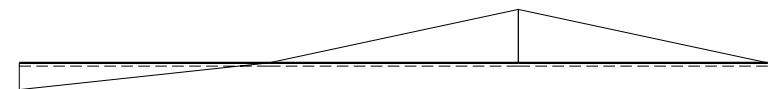




$\frac{2 F b^3}{E J}$



$\frac{1}{2} F$



$\frac{1}{2} F b$

AZIONI INTERNE (coordinate locali)

$$N_{AB} = 0$$

$$T_{AB} = -F$$

$$M_{AB} = Fb - Fx$$

$$N_{BC} = 0$$

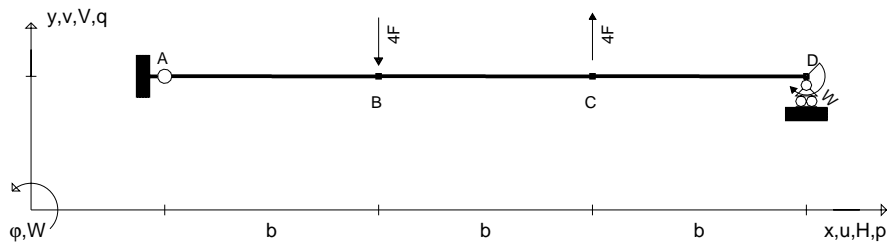
$$T_{BC} = -2F$$

$$M_{BC} = -2Fx$$

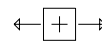
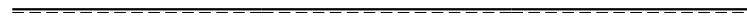
$$N_{CD} = 0$$

$$T_{CD} = 2F$$

$$M_{CD} = -2Fb + 2Fx$$

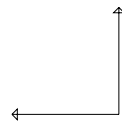
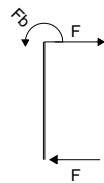
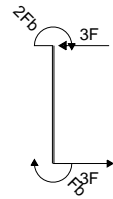
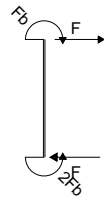


$$\begin{array}{lll}
 V_B = -4F & W_D = -W = -Fb & EJ_{BC} = EJ \\
 V_C = 4F & EJ_{AB} = EJ & EJ_{CD} = EJ
 \end{array}$$

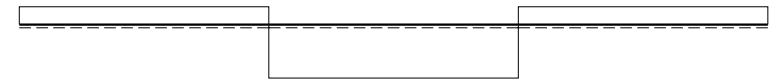


Verso effettivo dei carichi riportato nel disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi delle azioni interne nelle aste.
 Esprimere le funzioni delle azioni interne nelle aste.
 J_{AB} x_{AB} ϑ_{AB} riferimento locale asta AB con origine in A.

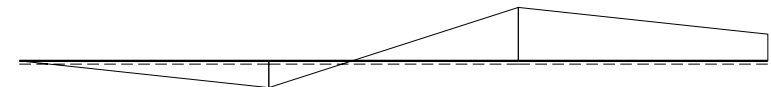
MANTENERE I RISULTATI IN FORMA FRAZIONARIA



$1.5 Fb^3/EJ$



$\uparrow + \downarrow$
 $13 F$



$\curvearrowright + \curvearrowleft$
 $12 Fb$

AZIONI INTERNE (coordinate locali)

$$N_{AB} = 0$$

$$T_{AB} = F$$

$$M_{AB} = Fx$$

$$N_{BC} = 0$$

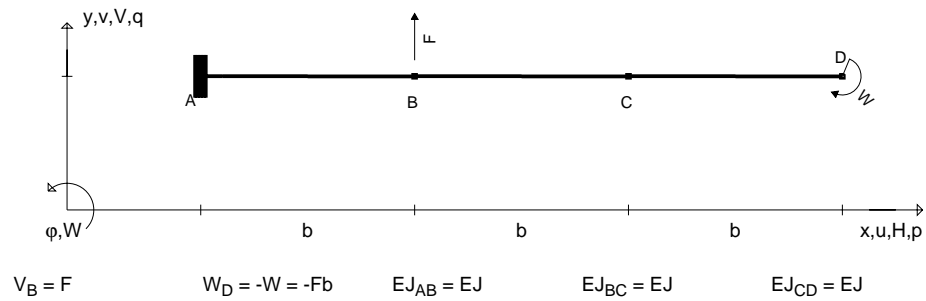
$$T_{BC} = -3F$$

$$M_{BC} = Fb - 3Fx$$

$$N_{CD} = 0$$

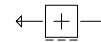
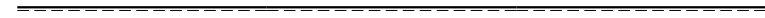
$$T_{CD} = F$$

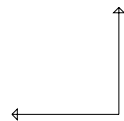
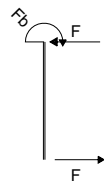
$$M_{CD} = -2Fb + Fx$$



Verso effettivo dei carichi riportato nel disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi delle azioni interne nelle aste.
 Esprimere le funzioni delle azioni interne nelle aste.
 J_{AB} x_{AB} ϑ_{AB} riferimento locale asta AB con origine in A.

MANTENERE I RISULTATI IN FORMA FRAZIONARIA

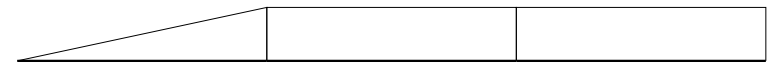




$6 F_b^3 / EJ$



$\uparrow + \downarrow$
 $1 F$



$\curvearrowright + \curvearrowleft$
 $1 F_b$

AZIONI INTERNE (coordinate locali)

$$N_{AB} = 0$$

$$T_{AB} = -F$$

$$M_{AB} = -Fx$$

$$N_{BC} = 0$$

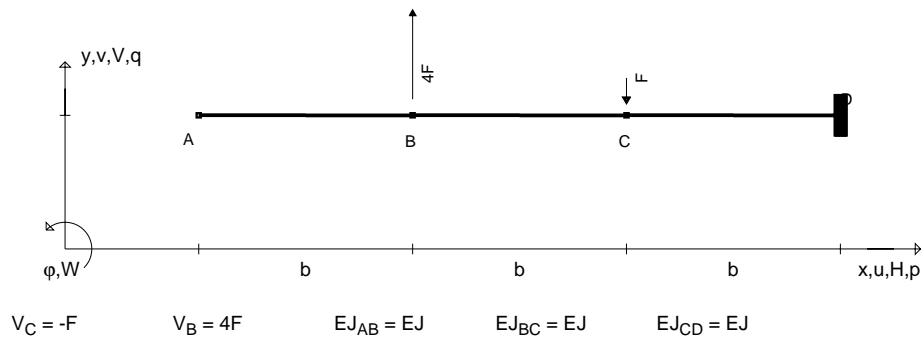
$$T_{BC} = 0$$

$$M_{BC} = -Fb$$

$$N_{CD} = 0$$

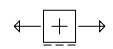
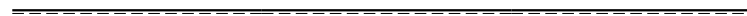
$$T_{CD} = 0$$

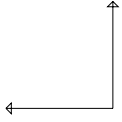
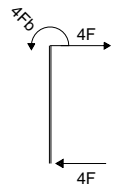
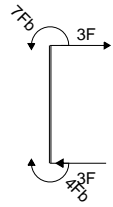
$$M_{CD} = -Fb$$



Verso effettivo dei carichi riportato nel disegno.
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MANTENERE I RISULTATI IN FORMA FRAZIONARIA

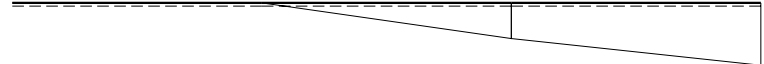




$40 Fb^3/EJ$



$4 F$



$16 Fb$

AZIONI INTERNE (coordinate locali)

$$N_{AB} = 0$$

$$T_{AB} = 0$$

$$M_{AB} = 0$$

$$N_{BC} = 0$$

$$T_{BC} = 4F$$

$$M_{BC} = 4Fx$$

$$N_{CD} = 0$$

$$T_{CD} = 3F$$

$$M_{CD} = 4Fb + 3Fx$$

CORSO DI T. E P. DI COSTRUZIONI E STRUTTURE

A.A. 2003-04

I Prova di Recupero – 14.07.2004

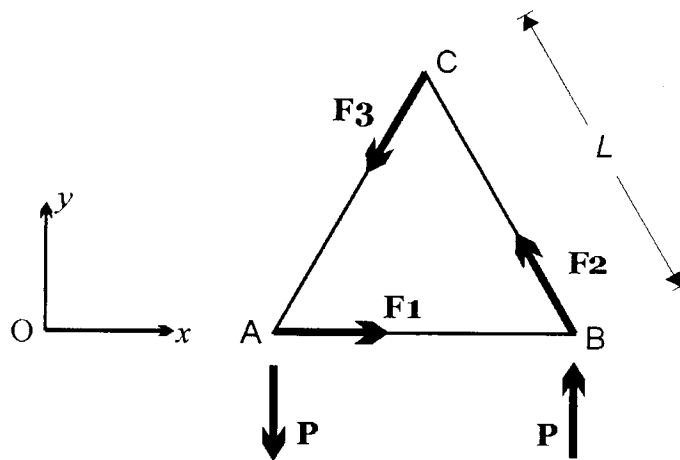
Nota: I risultati numerici (in forma frazionaria o con 3 cifre decimali) vanno riportati su questo stesso foglio, nei riquadri predisposti.

Allievo:.....	Matricola:.....
---------------	-----------------

FILA 1

Esercizio n.1 (18 punti)

Tre forze di ugual modulo $F = 200\text{N}$ sono applicate lungo i lati di un triangolo equilatero di lato $L = 200\text{ mm}$.

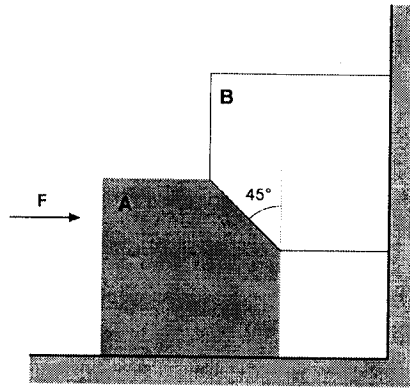


- Determinare le componenti delle 3 forze F_1, F_2, F_3 , nel sistema di riferimento Oxy .
- Determinare le componenti della risultante delle tre forze.
- Determinare il momento generato dalle 3 forze rispetto ai punti A, B e C.
- Determinare il modulo P delle due forze P necessario a garantire l'equilibrio.

$F_{1,x} =$	$F_{2,x} =$	$F_{3,x} =$
$F_{1,y} =$	$F_{2,y} =$	$F_{3,y} =$
$R_x =$	$R_y =$	$P =$
$M_A =$	$M_B =$	$M_C =$

Esercizio n.2 (12 punti)

Nel sistema in figura le masse dei due blocchi sono $m_A = 42 \text{ kg}$ e $m_B = 50 \text{ kg}$.
I due blocchi, assimilabili a punti materiali, sono lisci e trasmettono quindi azioni solo in direzione ortogonale alle superfici di contatto.

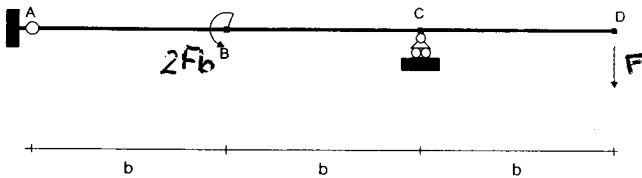


Determinare il valore dei moduli della forza F necessaria per garantire l'equilibrio del sistema, dell'azione mutua R_{AB} che si scambiano i due blocchi, della reazione del pavimento R_A sul blocco A e della reazione della parete R_B sul blocco B.

$F = \dots\dots\dots$; $R_{AB} = \dots\dots\dots$; $R_A = \dots\dots\dots$; $R_B = \dots\dots\dots$

Esercizio n.3 (15 punti)

Per la struttura in figura, riportare le reazioni vincolari e gli andamenti ed i diagrammi delle azioni interne M, N, T.



$\left[\begin{array}{c} \leftarrow + \rightarrow \\ N \end{array} \right]$ _____

$\left[\begin{array}{c} \uparrow + \downarrow \\ T \end{array} \right]$ _____

$\left[\begin{array}{c} \curvearrowright + \curvearrowleft \\ M \end{array} \right]$ _____

$V_A = \dots\dots\dots$; $H_A = \dots\dots\dots$; $V_C = \dots\dots\dots$

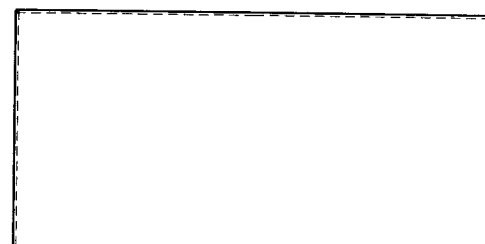
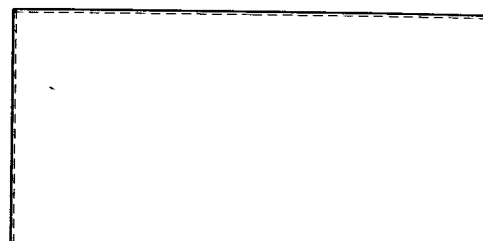
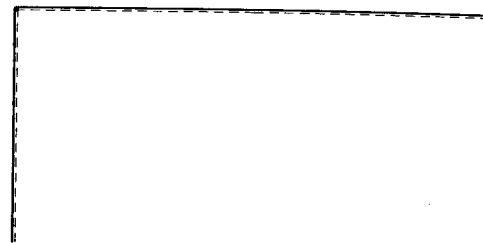
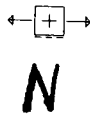
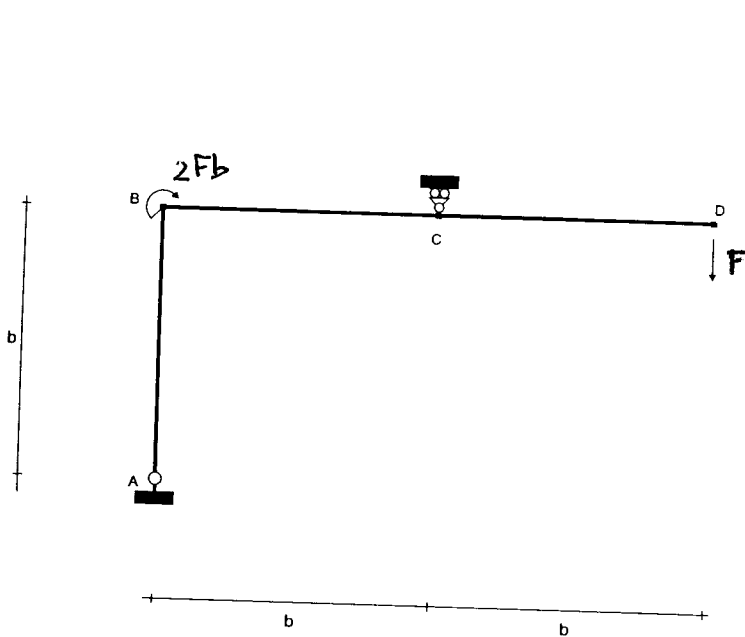
$N_{AB} =$
 $T_{AB} =$
 $M_{AB} =$

$N_{BC} =$
 $T_{BC} =$
 $M_{BC} =$

$N_{CD} =$
 $T_{CD} =$
 $M_{CD} =$

Esercizio n.4 (15 punti)

Per la struttura in figura, riportare le reazioni vincolari e gli andamenti ed i diagrammi delle azioni interne M, N, T.



$$N_{AB} =$$

$$N_{BC} =$$

$$N_{CD} =$$

$$T_{AB} =$$

$$T_{BC} =$$

$$T_{CD} =$$

$$M_{AB} =$$

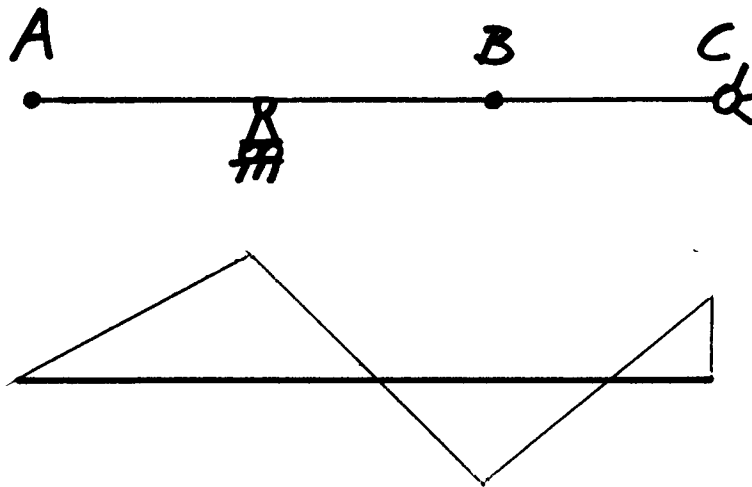
$$M_{BC} =$$

$$M_{CD} =$$

$V_A = \dots\dots\dots; H_A = \dots\dots\dots; V_C = \dots\dots\dots$

Esercizio n.5 (3 punti)

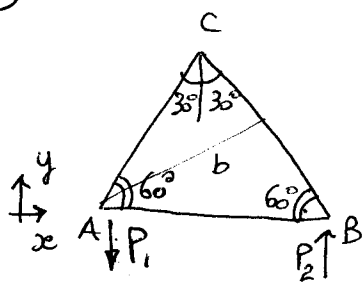
Si consideri la struttura data in figura con i rispettivi vincoli. Si indichi quali azioni e' necessario applicare nei punti A, B e C per ottenere la distribuzione di momento flettente data in figura.



1	
2	
3	
4	
5	
TOT	

I RECUPERO DEL 14.07.04

①



$$A = (0, 0)$$

$$B = (L, 0)$$

$$C = \left(\frac{L}{2}, \frac{\sqrt{3}}{2}L\right)$$

$$b = \frac{\sqrt{3}}{2}L$$

$$\vec{F}_1 = F\vec{i}$$

$$\vec{F}_2 = -F\cos 60^\circ \vec{i} + F\sin 60^\circ \vec{j} = -\frac{1}{2}F\vec{i} + \frac{\sqrt{3}}{2}F\vec{j}$$

$$\vec{F}_3 = -F\sin 30^\circ \vec{i} - F\cos 60^\circ \vec{j} = -\frac{1}{2}F\vec{i} - \frac{\sqrt{3}}{2}F\vec{j}$$

$$1. \quad F_{1x} = F \quad F_{2x} = -\frac{1}{2}F \quad F_{3x} = -\frac{1}{2}F$$

$$F_{1y} = 0 \quad F_{2y} = +\frac{\sqrt{3}}{2}F \quad F_{3y} = -\frac{\sqrt{3}}{2}F$$

$$2. \quad \vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\vec{R} = R_x\vec{i} + R_y\vec{j}$$

$$R_x = F_{1x} + F_{2x} + F_{3x} = F - \frac{1}{2}F - \frac{1}{2}F = 0$$

$$R_y = F_{1y} + F_{2y} + F_{3y} = 0 + \frac{\sqrt{3}}{2}F - \frac{\sqrt{3}}{2}F = 0$$

$$\} \Rightarrow \vec{R} = \vec{0} !$$

$$3. \quad M_{z(A)} = +F_2 b = +F_{2y} \cdot L = \frac{\sqrt{3}}{2}FL$$

$$\curvearrowright M_{z(B)} = +F_3 b = -F_{3y} \cdot \frac{L}{2} - F_{3x} \cdot \frac{\sqrt{3}}{2}L = \frac{\sqrt{3}}{2}F \cdot \frac{L}{2} + \frac{F}{2} \cdot \frac{\sqrt{3}}{2}L = 2 \cdot \frac{\sqrt{3}}{4}FL = \frac{\sqrt{3}}{2}FL$$

$$\curvearrowright M_{z(C)} = +F_1 \cdot b = F_{1x} \cdot \frac{\sqrt{3}}{2}L = \frac{\sqrt{3}}{2}FL$$

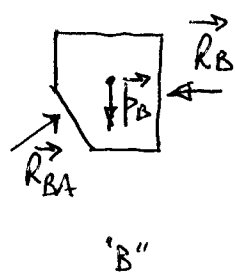
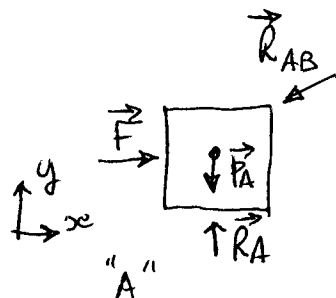
$$4. \quad \vec{P}_2 = P_2\vec{j} \quad \vec{P}_1 = -P_1\vec{j}$$

$$\vec{R} = \vec{0} \Leftrightarrow \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{P}_1 + \vec{P}_2 = \vec{0} \quad \vec{P}_1 + \vec{P}_2 = \vec{0} \quad \vec{P}_1 = -\vec{P}_2$$

$$M_{z(A)} = 0 \Leftrightarrow F_2 b + P_2 \cdot L = 0 \quad P_2 = -F_2 \frac{b}{L} = -F_2 \cdot \frac{\sqrt{3}}{2} \frac{L}{L} = -F \frac{\sqrt{3}}{2}$$

$$|\vec{P}_2| = |\vec{P}_1| = |\vec{P}| = +F \frac{\sqrt{3}}{2}, \quad F = |\vec{F}|$$

② Diagrammi di corpo libero:



$$\vec{P}_B = -m_B g \vec{j}$$

$$\vec{R}_B = -R_B \vec{i}$$

$$\vec{R}_{BA} = +R_{AB} \left(\frac{\sqrt{2}}{2} \vec{i} + \frac{\sqrt{2}}{2} \vec{j} \right)$$

NB: $\vec{R}_{BA} = -\vec{R}_{AB}$ perché sono forze interne \Rightarrow vale principio di azione e reazione ①

$$\vec{F} = F\vec{i}$$

$$\vec{P}_A = -m_A g \vec{j}$$

$$\vec{R}_A = +R_A \vec{j}$$

$$\vec{R}_{AB} = -R_{AB} \left(\frac{\sqrt{2}}{2} \vec{i} + \frac{\sqrt{2}}{2} \vec{j} \right)$$

Equilibrio p.m. "A":

$$\begin{cases} R_x^{(A)} = 0 & F - R_{AB} \frac{\sqrt{2}}{2} = 0 \quad [1] \\ R_y^{(A)} = 0 & -m_A g + R_A - R_{AB} \frac{\sqrt{2}}{2} = 0 \quad [2] \end{cases}$$

Equilibrio p.m. "B":

$$\begin{cases} R_x^{(B)} = 0 & R_{AB} \frac{\sqrt{2}}{2} - R_B = 0 \quad [3] \\ R_y^{(B)} = 0 & R_{AB} \frac{\sqrt{2}}{2} - m_B g = 0 \quad [4] \end{cases}$$

Per [4]: $R_{AB} = m_B g \cdot \frac{2}{\sqrt{2}} = \sqrt{2} m_B g$

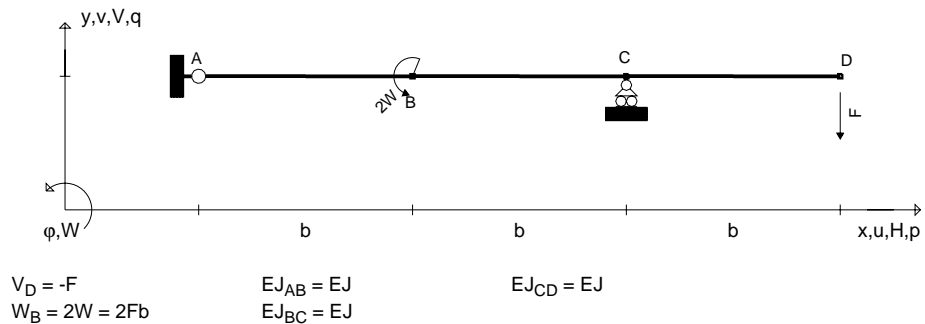
Per [3]: $R_B = \frac{\sqrt{2}}{2} (\sqrt{2} m_B g) = m_B g$

Per [2]: $R_A = m_A g + m_B g \sqrt{2} \cdot \frac{\sqrt{2}}{2} = (m_A + m_B) g$

Per [1]: $F = R_{AB} \frac{\sqrt{2}}{2} = \sqrt{2} m_B g \cdot \frac{\sqrt{2}}{2} = m_B g$

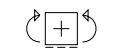
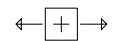
Esercizio 1		Testo 1	Testo 2	Testo 3	Testo 4
F	[N]	200	400	1400	1200
L	[mm]	200	100	400	300
F_1x	[N]	200.000	400.000	1400.000	1200.000
F_1y	[N]	0.000	0.000	0.000	0.000
F_2x	[N]	-100.000	-200.000	-700.000	-600.000
F_2y	[N]	173.205	346.410	1212.436	1039.230
F_3x	[N]	-100.000	-200.000	-700.000	-600.000
F_3y	[N]	-173.205	-346.410	-1212.436	-1039.230
R_x	[N]	0.000	0.000	0.000	0.000
R_y	[N]	0.000	0.000	0.000	0.000
 P 	[N]	173.205	346.410	1212.436	1039.230
M_A	[N·mm]	34641.016	34641.016	484974.226	311769.145
M_B	[N·mm]	34641.016	34641.016	484974.226	311769.145
M_C	[N·mm]	34641.016	34641.016	484974.226	311769.145

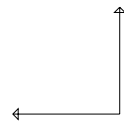
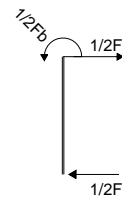
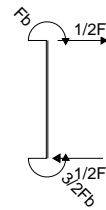
Esercizio 2		Testo 1	Testo 2	Testo 3	Testo 4
m_A	[kg]	42	24	124	142
m_B	[kg]	50	30	80	60
g	[m/s ²]	9.81	9.81	9.81	9.81
F	[N]	490.500	294.300	784.800	588.600
R_A	[N]	902.520	529.740	2001.240	1981.620
R_B	[N]	490.500	294.300	784.800	588.600
R_AB	[N]	693.672	416.203	1109.875	832.406



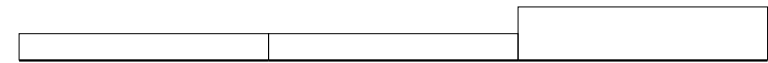
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MANTENERE I RISULTATI IN FORMA FRAZIONARIA

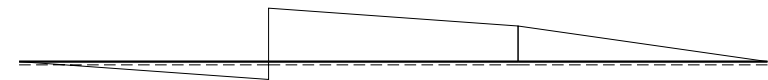




$2.5 Fb^3/EJ$



$1 F$



$1.5 Fb$

AZIONI INTERNE (coordinate locali)

$$N_{AB} = 0$$

$$T_{AB} = 1/2F$$

$$M_{AB} = 1/2Fx$$

$$N_{BC} = 0$$

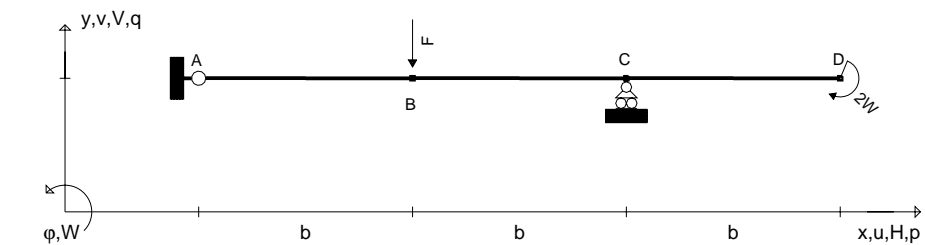
$$T_{BC} = 1/2F$$

$$M_{BC} = -3/2Fb + 1/2Fx$$

$$N_{CD} = 0$$

$$T_{CD} = F$$

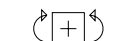
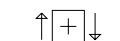
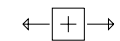
$$M_{CD} = -Fb + Fx$$

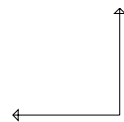
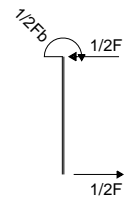
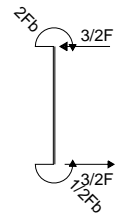


$V_B = -F$ $EJ_{AB} = EJ$ $EJ_{CD} = EJ$
 $W_D = -2W = -2Fb$ $EJ_{BC} = EJ$

Verso effettivo dei carichi riportato nel disegno.
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 J_{AB} x_{AB} ϑ_{AB} riferimento locale asta AB con origine in A.

MANTENERE I RISULTATI IN FORMA FRAZIONARIA

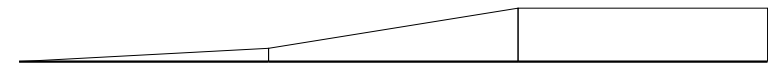




$5 Fb^3/EJ$



$1.5 F$



$2 Fb$

AZIONI INTERNE (coordinate locali)

$$N_{AB} = 0$$

$$T_{AB} = -1/2F$$

$$M_{AB} = -1/2Fx$$

$$N_{BC} = 0$$

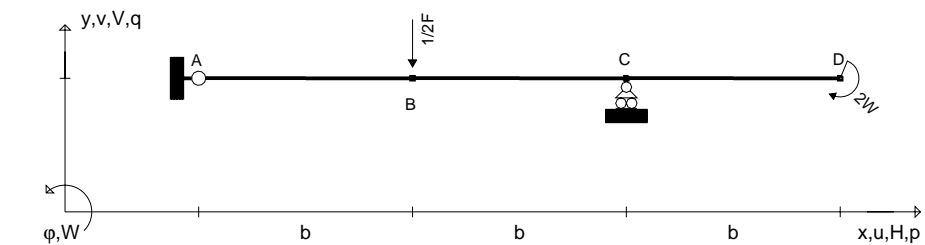
$$T_{BC} = -3/2F$$

$$M_{BC} = -1/2Fb - 3/2Fx$$

$$N_{CD} = 0$$

$$T_{CD} = 0$$

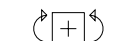
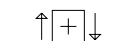
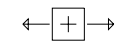
$$M_{CD} = -2Fb$$

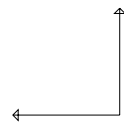
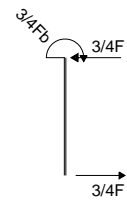
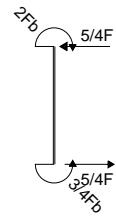


$V_B = -1/2F$ $EJ_{AB} = EJ$ $EJ_{CD} = EJ$
 $W_D = -2W = -2Fb$ $EJ_{BC} = EJ$

Verso effettivo dei carichi riportato nel disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi delle azioni interne nelle aste.
 Esprimere le funzioni delle azioni interne nelle aste.
 J_{AB} x_{AB} ϑ_{AB} riferimento locale asta AB con origine in A.

MANTENERE I RISULTATI IN FORMA FRAZIONARIA

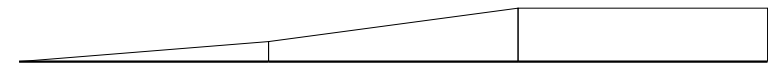




$5 Fb^3/EJ$



$1.2 F$



$2 Fb$

AZIONI INTERNE (coordinate locali)

$$N_{AB} = 0$$

$$T_{AB} = -3/4F$$

$$M_{AB} = -3/4Fx$$

$$N_{BC} = 0$$

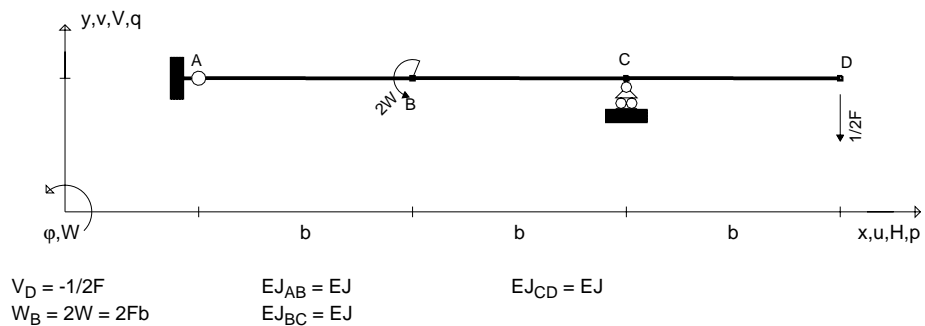
$$T_{BC} = -5/4F$$

$$M_{BC} = -3/4Fb - 5/4Fx$$

$$N_{CD} = 0$$

$$T_{CD} = 0$$

$$M_{CD} = -2Fb$$

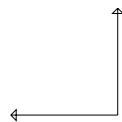
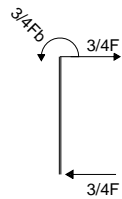
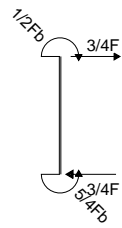


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 J_{AB} x_{AB} ϑ_{AB} riferimento locale asta AB con origine in A.

MANTENERE I RISULTATI IN FORMA FRAZIONARIA

Blank area for drawing internal force diagrams, with three horizontal lines and three sign convention symbols:

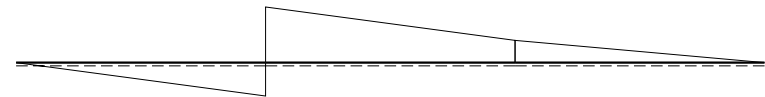
- Horizontal line 1:
- Horizontal line 2:
- Horizontal line 3:



$1.5 Fb^3/EJ$



$0.6 F$



$1.2 Fb$

AZIONI INTERNE (coordinate locali)

$$N_{AB} = 0$$

$$T_{AB} = 3/4F$$

$$M_{AB} = 3/4Fx$$

$$N_{BC} = 0$$

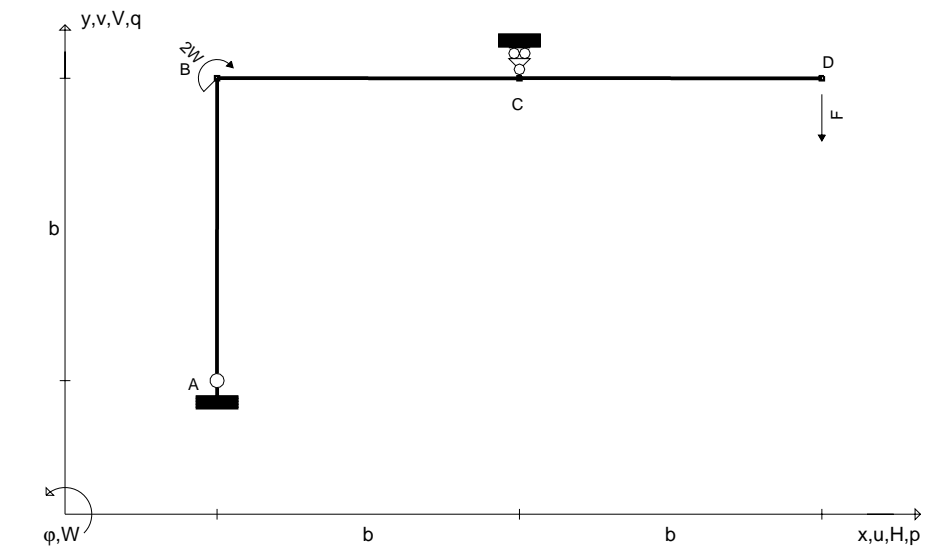
$$T_{BC} = 3/4F$$

$$M_{BC} = -5/4Fb + 3/4Fx$$

$$N_{CD} = 0$$

$$T_{CD} = 1/2F$$

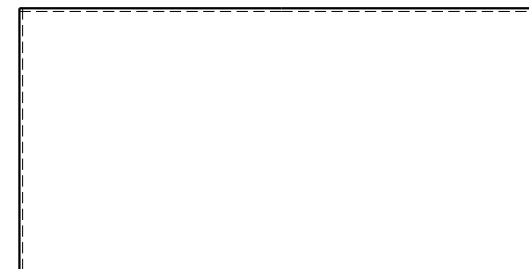
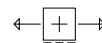
$$M_{CD} = -1/2Fb + 1/2Fx$$

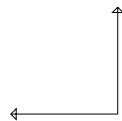
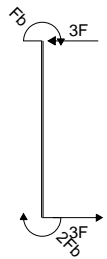
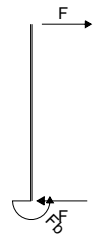


$V_D = -F$ $EJ_{AB} = EJ$ $EJ_{CD} = EJ$
 $W_B = -2W = -2Fb$ $EJ_{BC} = EJ$

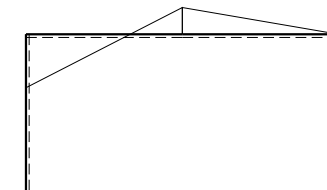
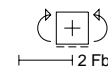
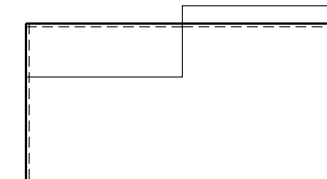
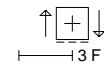
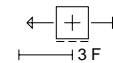
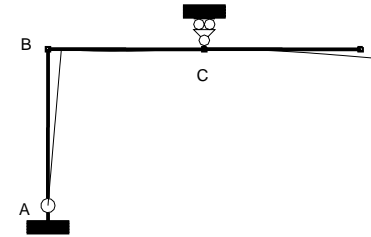
Verso effettivo dei carichi riportato nel disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi delle azioni interne nelle aste.
 Esprimere le funzioni delle azioni interne nelle aste.
 J_{AB} x_{AB} ϑ_{AB} riferimento locale asta AB con origine in A.

MANTENERE I RISULTATI IN FORMA FRAZIONARIA





$\frac{2 F b^3}{E J}$



AZIONI INTERNE (coordinate locali)

$$N_{AB} = 3F$$

$$T_{AB} = 0$$

$$M_{AB} = 0$$

$$N_{BC} = 0$$

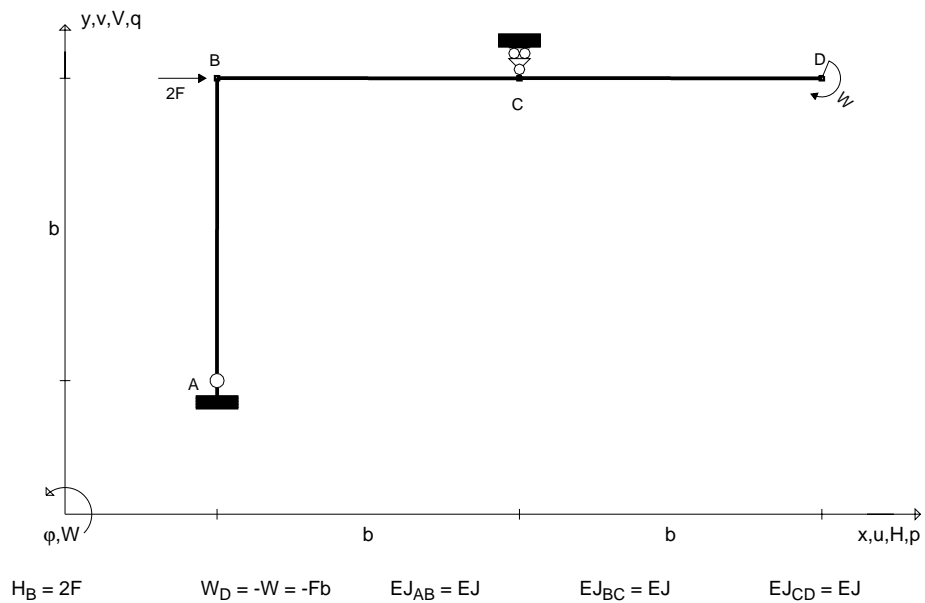
$$T_{BC} = -3F$$

$$M_{BC} = 2Fb - 3Fx$$

$$N_{CD} = 0$$

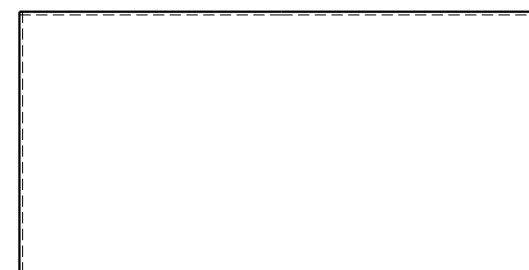
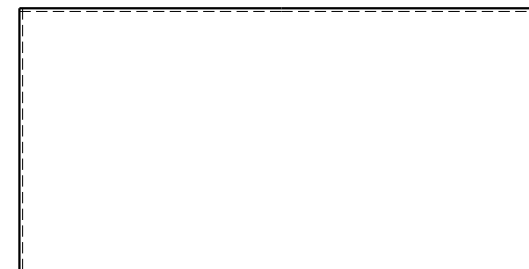
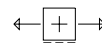
$$T_{CD} = F$$

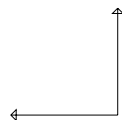
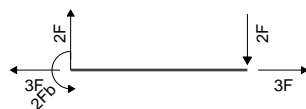
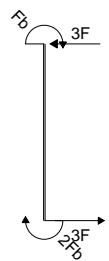
$$M_{CD} = -Fb + Fx$$



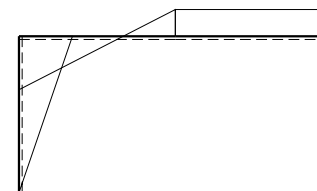
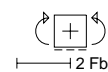
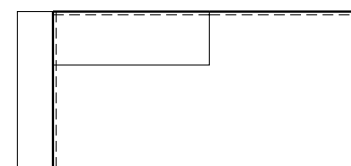
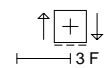
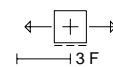
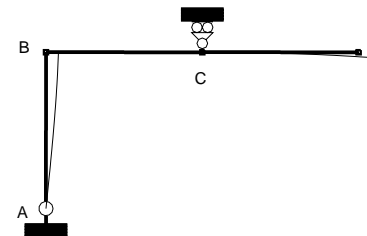
Verso effettivo dei carichi riportato nel disegno.
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 J_{AB} x_{AB} v_{AB} riferimento locale asta AB con origine in A.

MANTENERE I RISULTATI IN FORMA FRAZIONARIA





$5 Fb^3/EJ$



AZIONI INTERNE (coordinate locali)

$$N_{AB} = 3F$$

$$T_{AB} = 2F$$

$$M_{AB} = 2Fx$$

$$N_{BC} = 0$$

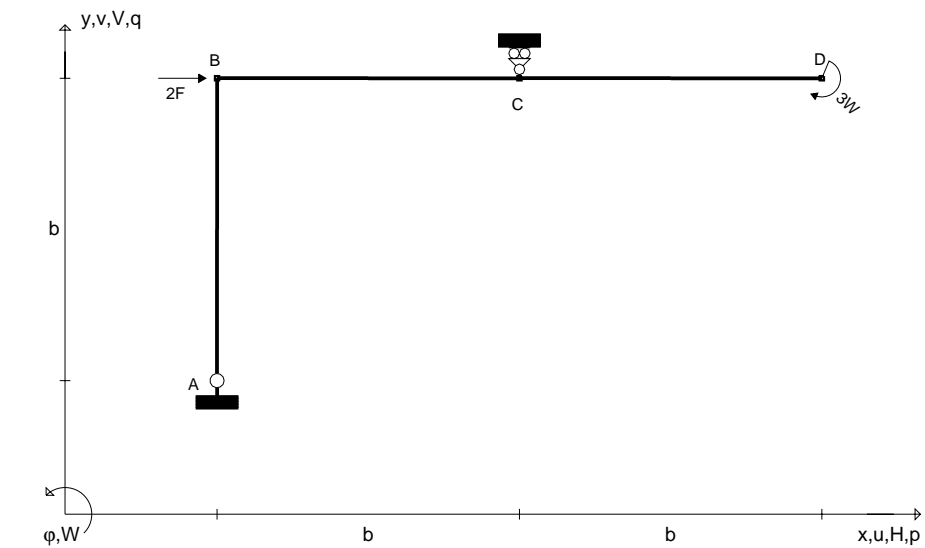
$$T_{BC} = -3F$$

$$M_{BC} = 2Fb - 3Fx$$

$$N_{CD} = 0$$

$$T_{CD} = 0$$

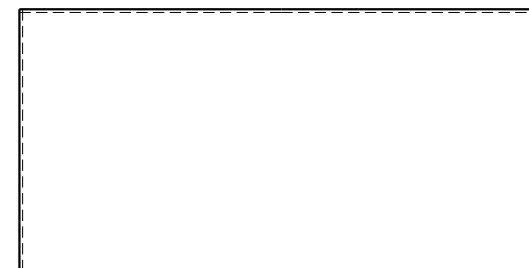
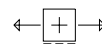
$$M_{CD} = -Fb$$

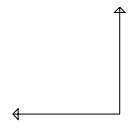
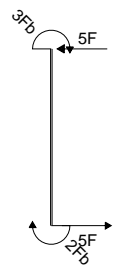
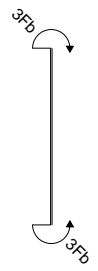


$H_B = 2F$ $EJ_{AB} = EJ$ $EJ_{CD} = EJ$
 $W_D = -3W = -3Fb$ $EJ_{BC} = EJ$

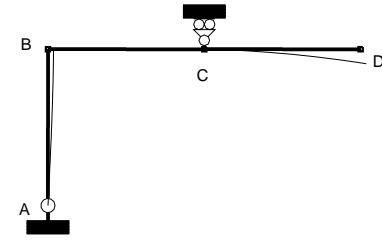
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 J_{AB} x_{AB} ϑ_{AB} riferimento locale asta AB con origine in A.

MANTENERE I RISULTATI IN FORMA FRAZIONARIA





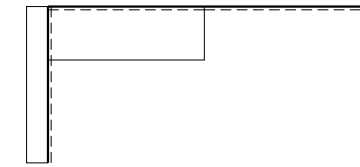
$8 Fb^3/EJ$



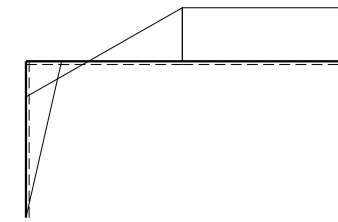
$15 F$



$15 F$



$13 Fb$



AZIONI INTERNE (coordinate locali)

$$N_{AB} = 5F$$

$$T_{AB} = 2F$$

$$M_{AB} = 2Fx$$

$$N_{BC} = 0$$

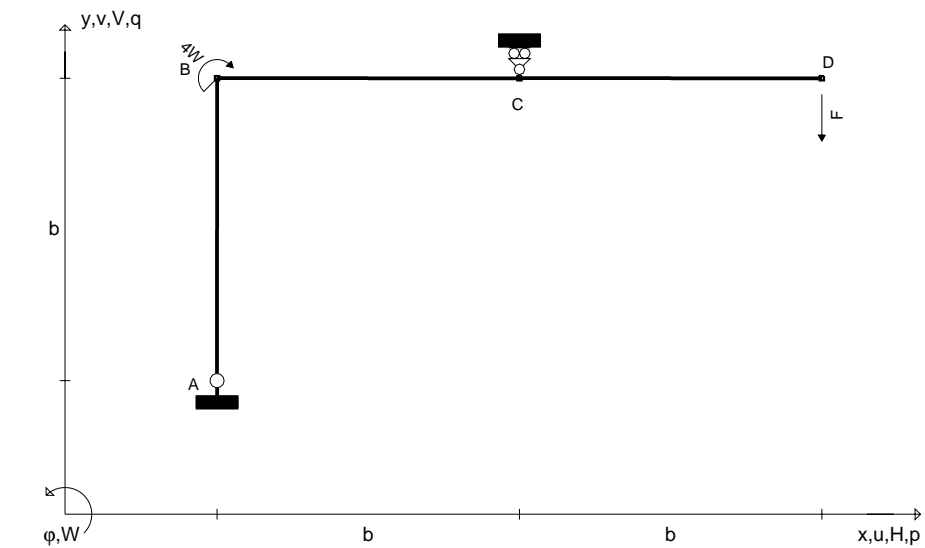
$$T_{BC} = -5F$$

$$M_{BC} = 2Fb - 5Fx$$

$$N_{CD} = 0$$

$$T_{CD} = 0$$

$$M_{CD} = -3Fb$$



$$V_D = -F$$

$$W_B = -4W = -4Fb$$

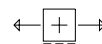
$$EJ_{AB} = EJ$$

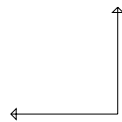
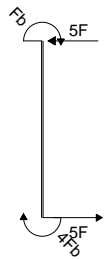
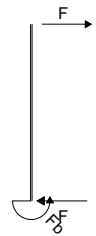
$$EJ_{BC} = EJ$$

$$EJ_{CD} = EJ$$

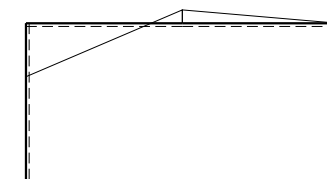
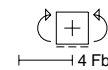
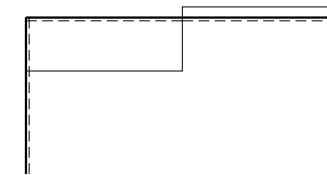
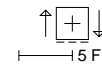
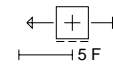
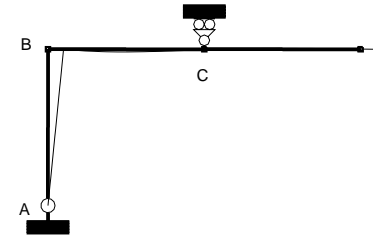
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MANTENERE I RISULTATI IN FORMA FRAZIONARIA





$4 Fb^3/EJ$



AZIONI INTERNE (coordinate locali)

$$N_{AB} = 5F$$

$$T_{AB} = 0$$

$$M_{AB} = 0$$

$$N_{BC} = 0$$

$$T_{BC} = -5F$$

$$M_{BC} = 4Fb - 5Fx$$

$$N_{CD} = 0$$

$$T_{CD} = F$$

$$M_{CD} = -Fb + Fx$$